Determining nuclear temperature in heavy-ion collisions

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The Phases of Nuclear Matter

Potential vs Distance

- Potential $10^{-10}$ m

Temperature vs Excitation Energy per Molecule [meV]

- Water
- Ice
- Water Vapor

Potential vs Distance

- Potential $10^{-15}$ m

Temperature vs Excitation Energy per Nucleon [MeV]
Probing the Nuclear Liquid-Gas Phase Transition


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Caloric Curve:

Au+Au, 600 MeV/u

12C, 18O+natAg, 197Au, 30–84 MeV/u

22Ne+181Ta, 8 MeV/u
There is a mass dependence clearly shown in the Caloric Curves
Outline

1. Introduction
2. Theoretical model
3. Results and discussion
4. Conclusions and outlooks
1. Introduction

Definition of Temperature

1. Statistical mechanics:
   with fixed number of particles $N$ at an energy $E$

\[
\frac{1}{T} = \frac{\partial S(E, N_{\text{part}})}{\partial E} = \frac{\partial \ln \rho(E, N_{\text{part}})}{\partial E} \]

2. The kinetic theory of gases:
   In a classical ideal gas, the temperature is related to its average kinetic energy
   \[\langle E_k \rangle = \text{number of degree of freedom} \times \frac{1}{2}k_B T\]
Heavy ion collisions at intermediate and high energies

Projectile $\rightarrow$ Target

$T=?$

dense, hot, and asymmetric nuclear matter

Detectors

Equation of State Of Nuclear Matter

$E(\rho,T,\delta)=?$
The concept of temperature has been used in nuclear systems seventy years ago.

- From **compound nuclei** ($\rho \approx \rho_0$, $T \approx 1-2$ MeV),
- **hot nuclei** ($\rho \approx \rho_0$, $T > 5$ MeV),
- **highly excited nuclei** ($\rho \approx 3\rho_0$, $T > 5$ MeV),
- asymmetrical highly excited nuclei ($\rho \approx 3\rho_0$, $T > 5$ MeV, $\delta > 0$)

- **Nuclear equation of state (EOS)**

  $\rho \neq \rho_0$, $T > 0$, $\delta > 0$

  $E(\rho, T, \delta) = \text{?}$, **How to determine T in theory?**
Thermometer determination

- **Kinetic approaches**, Based on the canonical ensemble
  - Slope thermometer
  - Fluctuation temperature
    - S. Wuenschel et al., Nuclear Physics A 843, 1 (2010).

- **Population approaches**, Based on the grand-canonical ensemble,
  - Double ratios of isotopic yields
  - Population of excited states
  - Isobaric yields from a given source
Kinetic energy approaches

Originally proposed in 1937 in case of n-induced reactions (Maxwell-Boltzmann distribution)

$$\frac{dY}{dE_{\text{kin}}} = f(E_{\text{kin}}) \exp[-\frac{E_{\text{kin}}}{T}]$$

Slope thermometer
Westfall, PLB 116, 118 (1982)
Jacak et al., PRL 51, 1846 (1983)

Fluctuation thermometer
Wuenschel et al., NPA 843, 1 (2010)
Slope thermometer

The slope temperature is extracted by fitting the slope of the particle spectra

The spectra shape can be Influenced by collective dynamical effects

Westfall et al, PLB 116, 118 (1982)

Jacak et al., PRL 51, 1846 (1983)
Fluctuation thermometer

Using the momentum fluctuation, the nuclear temperature can also be derived.

\[ \sigma^2 = \langle Q_z^2 \rangle - \langle Q_z \rangle^2 \]

\[ \langle Q_z^2 \rangle = \int d^3 p \left( 2 P_Z^2 - P_T^2 \right)^2 f(p) \]

\[ \sigma^2 = 12m_0^2T^2 \sum_i (\xi_i A_i)^2 \]

Wuenschel et al., NPA 843 (2010) 1
• Population of excited states

The ratio of the populations of 2 states

\[ R = \frac{2j_\mu + 1}{2j_l + 1} \exp\left[-\frac{\Delta E}{T}\right] \]

Correction: decay, final-state interaction,…

Morrissey et al., PLB148, 423 (1984)
Double ratios of isotopic yields

\[ \rho(A, Z) = \frac{N_{part}}{V} = \frac{A^{3/2} \cdot \omega(A, Z)}{\lambda^3} \cdot \exp \left( \frac{\mu(A, Z)}{T} \right) \]

Ratio between the 2 different emitted fragments

\[ \frac{Y(A, Z)}{Y(A', Z')} = \frac{\rho(A, Z)}{\rho(A', Z')} = \left( \frac{A}{A'} \right)^{3/2} \cdot \left( \frac{\lambda^3}{2} \right)^{A-A'} \]

\[ \cdot \frac{\omega(A, Z)}{\omega(A', Z')} \cdot \frac{\rho_{pF}}{\rho_{nF}} \cdot \frac{Z-Z'}{(A-Z)-(A'-Z')} \]

\[ \cdot \exp \left( \frac{B(A, Z) - B(A', Z')}{T} \right) \]

Temperature

\[ T = (\Delta B_1 - \Delta B_2) / \ln \left[ \frac{\left( \frac{Y(A_1, Z_1)/Y(A_1+1, Z_1+1)}{Y(A_2, Z_2)/Y(A_2+1, Z_2+1)} \right)}{\left( \frac{(A_1+1) \cdot A_2}{A_1 \cdot (A_2+1)} \right)^{3/2}} \right. \]

\[ \cdot \frac{\omega(A_1+1, Z_1+1) \cdot \omega(A_2, Z_2)}{\omega(A_1, Z_1) \cdot \omega(A_2+1, Z_2+1)} \]

Excitation Energy in Heavy Ion Collisions

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Abstract

With Hartree-Fock approximation, the relation between excitation energy and temperature has been obtained and discussed for both infinite nuclear matter and finite nucleus Pb\text{208}.


图1 不同密度下无穷大核物质和有限核 Pb\text{208} 的单核子热激发能随温度的变化。

(a) Φ(1n^{-2}) 0.05\text{MeV} \times 0.16 \text{MeV} 自由核子——
(b) Φ(1n^{-2}) 0.05\text{MeV} \times 0.16 \text{MeV} 自由核子——
A New Interpretation for ALADIN Caloric Curve

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Abstract Within the framework of Hartree-Fock theory with extended Skyrme effective interaction, the excitation energy as a function of temperature and density was investigated and used to analyse the ALADIN caloric curve. Our work began on the assumption that the temperature plateau of ALADIN caloric curve was resulted from the compression excitation energy. The theoretical calculations with this assumption were in good agreement with the ALADIN caloric curve, which indicates that our assumption is reasonable, i.e., the temperature plateau of ALADIN caloric curve is resulted from the compression excitation energy, and liquid-gas phase transition isn’t the only interpretation for the ALADIN caloric curve. Therefore, we provided a new interpretation for the ALADIN caloric curve.
Phase transitions, correlations and fluctuations of nuclear multifragmentation

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**Fig. 1a,b.** Pressure-density isotherms for nuclear matter (a) and $^{197}$Au (b). In both figures, the coexistence line (long dotted-dashed) is defined as the boundary of two phase regions in thermodynamic equilibrium and the spinodal lines (dotted-dashed) are defined as the isothermal incompressibility vanish

**Fig. 5a,b.** The scatter plots of the correlation between $\ln S_3$ and $\ln S_2$ a, and the correlation between $\ln (Z_{MAX})$ and $\ln S_2$ b, for the spinodal, the super heated liquid, the hot liquid regions and their mixing for $^{197}$Au at T=6 MeV
Nuclear thermometry

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Abstract. Different approaches for measuring nuclear temperatures are described. The quantitative results of different thermometer approaches are often not consistent. These differences are traced back to the different basic assumptions of the applied methods. Moreover, an overview of recent theoretical investigations is given, which study the quantitative influence of dynamical aspects of the nuclear-reaction process on the extracted apparent temperatures. The status of the present experimental and theoretical knowledge is reviewed. Guidelines for future investigations, especially concerning the properties of asymmetric nuclear matter, are given.

PACS. 24.60.-k Nuclear reaction: general; Statistical theory and fluctuations – 05.70.Fh Phase transitions: general studies – 25.70.-z Low and intermediate energy heavy-ion reactions – 21.10.Ma Level density
2. Theoretical Model

hot nuclear system \( \rightarrow \) excited pre-fragments \( \rightarrow \) final products

Multifragmentation

50 fm/c \( \rightarrow \) de-excitation \( \rightarrow \) 200 fm/c

\[ v=0.1-0.5c \]

Isospin-dependent Quantum Molecular Dynamics model \( \rightarrow \) statistical decay model (GEMINI)

THERMAL SHOCK

FREEZEOUT

SECONDARY EMISSION

EXPANSION

PRE-EQUILIBRIUM EMISSION

EQUILIBRIUM EMISSION ?

SEPARATION
**Isospin dependent quantum molecular dynamics model + Gemini**

- **mean field (corresponds to interactions)**

\[
U (\rho, \tau_z) = U^{loc} + U^{Yuk} + U^{Coul} + U^{Sym} + U^{MDI}
\]

- \(U^{loc}\): density dependent potential
- \(U^{Yuk}\): Yukawa (surface) potential
- \(U^{Coul}\): Coulomb energy
- \(U^{Sym}\): symmetry energy
- \(U^{MDI}\): momentum dependent interaction

- **two-body collisions + pauli blocking**
- **initialization**
- **coalescence model**
- **Gemini**
To check the model

Odd-even effect in heavy-ion collisions at intermediate energies

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(Received 7 December 2010; published 31 January 2011)

Charge distributions, multiplicities, and the energy spectra
3. Results and Discussion

Charge and $Z_{\text{bound}}$ distributions

$\langle M_{\text{IMF}} \rangle$ and $Z_{\text{max}}/Z_p \sim Z_{\text{bound}}/Z_p$

J. Su and F. S. Zhang, PRC 84 037601 (2011)
Isotopic Dependence of the Nuclear Caloric Curve

C. Sfienti,1 P. Adrich,1 T. Aumann,1 C. O. Bacri,2 T. Barczyk,3 R. Bassini,4 S. Bianchin,1 C. Boiano,4 A. S. Botvina,1,5 A. Boudard,6 J. Brzychczyk,3 A. Chbihi,7 J. Cibor,8 B. Czech,8 M. De Napoli,9 J.-É. Ducret,6 H. Emling,1 J. D. Frankland,7 M. Hellström,1 D. Henzlová,1 G. Immè,9 I. Iori,4,H H. Johansson,1 K. Kezzar,1 A. Lafiakh,6 A. Le Fèvre,1 E. Le Gentil,6 Y. Leifels,1 J. Lühning,1 J. Lukasik,1,8 W. G. Lynch,10 U. Lynen,1 Z. Majka,3 M. Mocko,10 W. F. J. Müller,1 A. Mykulyak,11 H. Orth,1 A. N. Otte,1 R. Palit,1 P. Pawlowski,8 A. Pullia,4 G. Raciti,9 E. Rapisarda,9 H. Sann,1,* C. Schwarz,1 H. Simon,1 K. Sümmerer,1 W. Trautmann,1 M. B. Tsang,10 G. Verde,10 C. Volant,6 M. Wallace,10 H. Weick,1 J. Wiechula,1 A. Wieloch,3 and B. Zwiegliński11

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FIG. 1 (color online). Acceptance corrected mean multiplicity $\langle M_{\text{imp}} \rangle$ of projectile fragments for $^{124}$Sn (circles), $^{124}$La (triangles), and $^{107}$Sn (open squares) beams of 600 A MeV on natSn targets as a function of Z_{bound} (left panel) and correlations of $\langle Z_{\text{max}} \rangle$ with Z_{bound} (both normalized with respect to the atomic number Z_{proj} of the projectile, right panel).
Isospin-dependent multifragmentation of relativistic projectiles


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The isotope temperatures show a smooth fall with increasing $Z_{\text{bound}} / Z_p$ for the reactions.

The $T_{\text{HeLi}}$ for the neutron-rich projectiles are larger than those for the neutron-poor projectiles.
With $A$ decreasing, $T$ increasing;

With $N/Z$ increasing, $T$ increasing;
\[ \frac{dY}{dE_{\text{kin}}} = f(E_{\text{kin}}) \exp\left[-\frac{E_{\text{kin}}}{T_{\text{slope}}}\right] \]

\[ T_{\text{slope}} > T_{\text{HeLi}} \]

Odeh et al. PRL, 84.4557 (2000)
How to distinguish the Fermi motion?

Kinetic energy including: thermal, Fermi motion, Collective flow + Coulomb

\[ E_{\text{tot}} = E_{\text{thermal}} + E_{\text{Fermi}} + E_{\text{flow}} + E_{\text{Coulomb}} \]

Maxwell

\[ f(p) \propto \exp\left(\frac{-p^2}{2mT}\right) \]

Fermi-Dirac

\[ f(p) \propto \frac{1}{1 + \exp\left(\frac{p^2}{2m} - \mu\right)/T} \]

Fermi distribution

\[ T_{\text{slop}} \text{ and } T'_{\text{slop}} \]

\[ \langle E_k \rangle = \int \frac{p^2}{2m} f(p) \frac{dp}{d\varepsilon} \]

\[ T_{\text{slope}} = \frac{A - A_f}{A - 1} \frac{2}{5} E_F \left(1 + \frac{5\pi^2 T^2}{12E_F^2}\right) \]

Su, Zhu, Xie, Zhang, PRC 85, 017604 (2012)
Temperatures of fragment kinetic energy spectra

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Multifragmentation reactions without large compression in the initial state (proton-induced reactions, reverse kinematics, projectile fragmentation) are examined, and it is verified quantitatively that the high temperatures obtained from fragment kinetic energy spectra and lower temperatures obtained from observables such as level population or isotope ratios can be understood in a common framework.

PACS number(s): 25.70.Pq

**FIG. 2.** Apparent temperature of fragment kinetic energy spectra (in units of the Fermi energy) as a function of the temperature, $T_{\text{in}}$ of the Fermi gas. Solid line: numerical solution of Eq. (8) inserted into Eq. (7). Dashed line: analytic approximation, Eq. (11).
Measuring the temperature of hot nuclear fragments

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Density and temperature of fermions from quantum fluctuations

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Nuclear temperatures from kinetic characteristics

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The kinematic characteristics of fragments are investigated using the isospin-dependent quantum molecular dynamics model accompanied by the statistical decay model GEMINI. The temperatures of single multifragmenting sources formed in those central heavy-ion collisions are extracted by two methods based on classical kinetic approaches. Differences between the slope temperature and the quadrupole temperature are discussed. Taking into account the Fermi-Dirac nature of finite nuclear systems, we derive the quantum temperatures. The quantum slope temperatures are lower than the isotope temperatures $T_{\text{HeLi}}$. The quantum quadrupole temperatures are higher than the isotope temperature.

FIG. 3. (Color online) Comparisons of slope temperatures extracted from kinetic energy spectra of different fragments between the present simulations (cal.) and experimental data (exp.) [9] for Au + Au central collisions at 35 MeV/nucleon.

FIG. 4. (Color online) Temperatures derived by different methods as a function of incident energies for central collisions of $^{129}$Xe + $^{129}$Sn and $^{197}$Au + $^{197}$Au at an incident energy of from 30 to 80 MeV/nucleon. Squares, isotopic thermometer temperatures; circles, quantum slope temperatures; triangles, quantum quadrupole temperatures.
To compare the 3 nuclear thermometers

Assumption: the traditional definition of temperature is suitable.
Systems: central heavy-ion collisions (Xe+Sn, Au+Au)
Energy: 30 - 80 MeV/u
Observable: difference between $T_{\text{HeLi}}$ and $T_{\text{slope}}$ ($T_{\text{flu}}$)

Maxwell distribution: $T > T_{\text{HeLi}}$
Fermi distribution: $T \sim T_{\text{HeLi}}$

Su, Zhu, Xie, Zhang, PRC 85, 017604 (2012)
\( T_{\text{Maxwell}} > T_{\text{Fermi}} \sim T_{\text{HeLi}} \)

<table>
<thead>
<tr>
<th></th>
<th>( T_{\text{slope}} ) (MeV)</th>
<th>( T_{\text{fluct}} ) (MeV)</th>
<th>( T'_{\text{slope}} ) (MeV)</th>
<th>( T'_{\text{fluct}} ) (MeV)</th>
<th>( T_{\text{HeLi}} ) (MeV)</th>
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<td>( ^{197}\text{Au}+^{197}\text{Au} )</td>
<td>35</td>
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For sodium Clusters Na$_n$
\[ Na^+_n(T_1) + jh\nu \Rightarrow Na^+_{n-x}(T_{evap.}) + xNa \]

\[ U_n(T_1) + jh\nu = U_{n-x}(T_{evap.}) + \sum_{i=1}^{x} D_i + \sum_{i=1}^{x} \varepsilon_i \]

\[ Na^+_n(T_2) + (j-1)h\nu \Rightarrow Na^+_{n-x}(T_{evap.}) + xNa \]

\[ U_n(T_2) + (j-1)h\nu = U_{n-x}(T_{evap.}) + \sum_{i=1}^{x} D_i + \sum_{i=1}^{x} \varepsilon_i \]

\[ U(T_1) + h\nu = U(T_2) \]

\[ U(T_1) + \delta U = U(T_2) \]

\[ C(T) = \frac{\partial U}{\partial T} \approx \frac{\delta U}{\delta T} = \frac{h\nu}{T_2 - T_1} \]
Heat capacity of Na$^{+}\text{}_{139}$ is plotted against the temperature.
4. Conclusions and outlooks

1. To verify different methods for determination of T: kinetic energy method, population of excited states, double ratios of isotopic yields

2. In each method, to know the reliability for different conditions

3. New methods are welcome for determination of T and it is still very far to get a proper definition of liquid-gas phase transitions in nuclear system

Thank you for your attention!