Magnetized Hot and Dense Quark Matter

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South America
N
GUYANA
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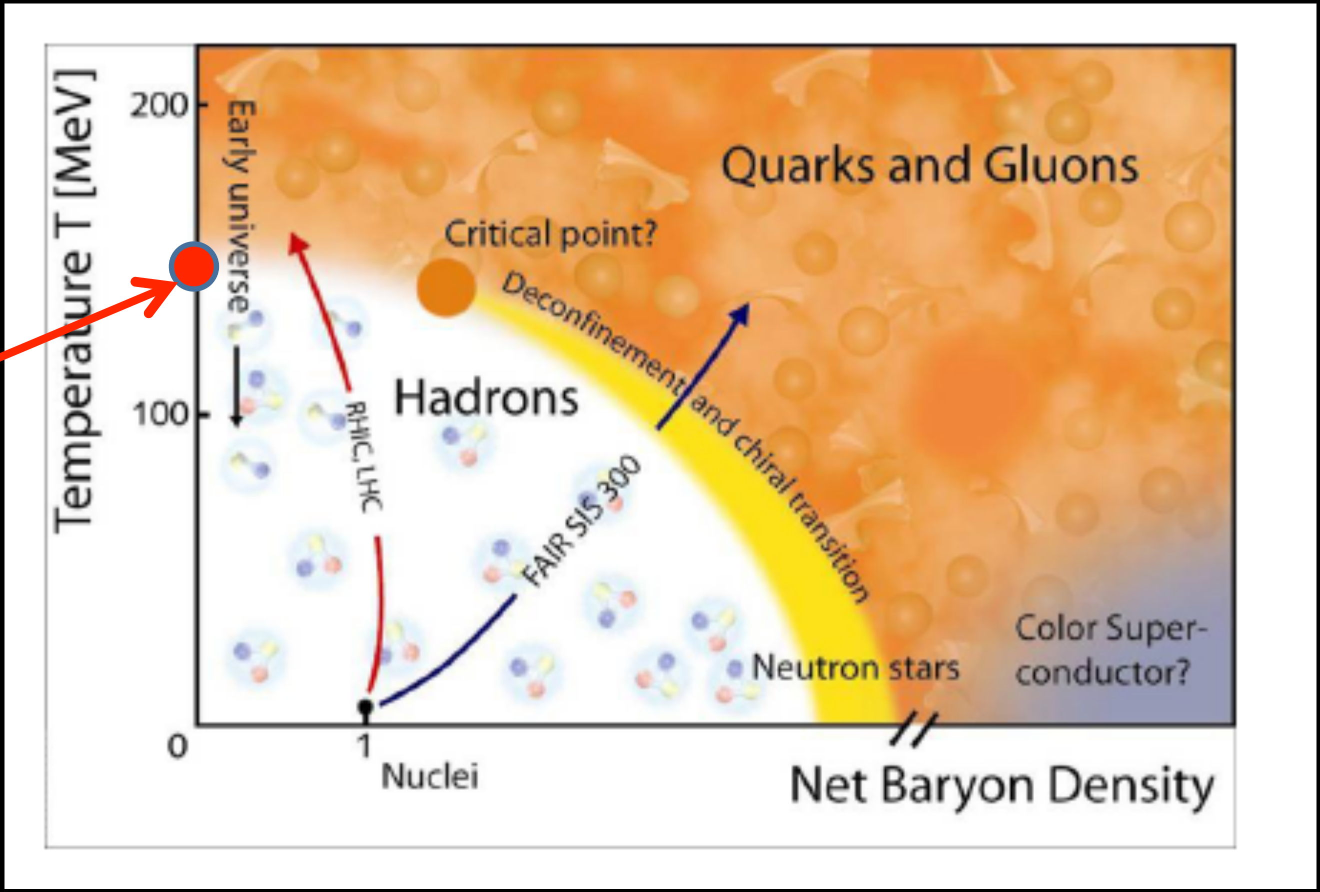
COLOMBIA
FRENCH
GUIANA
SURINAME
ECUADOR
BRAZIL
PERU
BOLIVIA
CHILE
PARAGUAY
URUGUAY
ARGENTINA
FALKLAND
ISLANDS
Atlantic
Ocean
Pacific
Ocean

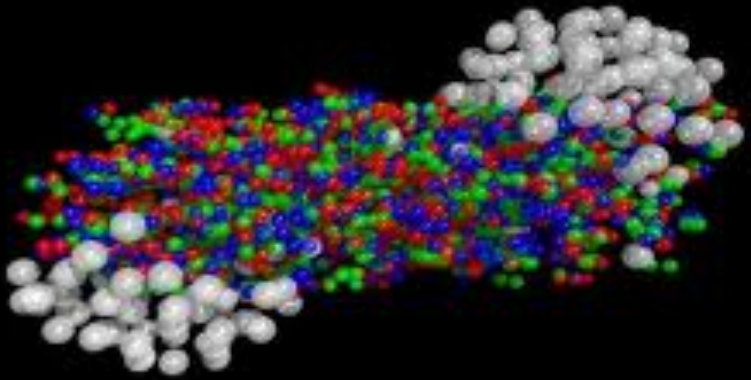

HOWDY !

(Theoretical) QCD Phase Diagram



Only sure about X-over at about T=170 MeV and μ=0. from LATTICE. At μ=0 PD obtained with effective models for QCD like LSM, NJL etc

WHY MAGNETIC FIELDS ??



NON central HIC

$eB \sim 5 - 15\ m_\pi^2 \sim 10^{19}$ G

MAGNETARS

$eB \sim 0.5\ m_\pi^2 \sim 10^{17}$ G

$\sim 10^{19}$ G at core ?

$1\ \mathrm{MeV}^2 \sim 10^{13}$ G

$e = 1/\sqrt{137}$

WHAT DO WE NEED TO STUDY PHASE TRANSITIONS

Statistical Mechanics, the Free-Energy

$$F[\mathbf{M}(x)] = \int d^3x \left[\mathcal{F}(\mathbf{M}(x)) + \frac{1}{2}K_L(\mathbf{M})(\nabla.\mathbf{M}(x))^2\right]$$

$\mathcal{F}$ (the bulk free energy density)

QFT, the Effective Action: generates ALL 1PI Greens functions

$$\Gamma[\sigma(x)] = \int d^4x \left[-V_{eff}(\sigma(x)) + \frac{1}{2}Z_\sigma(\partial_\mu\sigma)^2\right]$$

V_{eff} is known as the effective potential

At their minimum ($\sigma = \langle\sigma\rangle = \bar{\sigma}$ and $\mathbf{M} = \overline{\mathbf{M}}$) both V_{eff} and $\mathcal{F}$ represent the thermodynamical potential $\Omega = -P$.

The two-flavor NJL effective model for quarks

$$\mathcal{L} = \bar{\psi}\,(i\partial\!\!\!/ - m_c)\,\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right]$$

- Non-renormalizable since G : $1/(\mathrm{eV})^2$ => cut-off Λ (new "parameter")
- $m_c = 0$ => Chiral Symmetry dynamically broken by $G > G_c$ at T=0, μ=0
- No confinement in this version (can add Polyakov loop)
- No vector channel in this version

STANDARD PARAMETRiZATION:

$\Lambda \sim 600\mathrm{MeV}$ $\quad G\Lambda^2 \sim 2$ $\quad m_c \sim 5\mathrm{MeV}$

gives $\quad m_\pi = 135\ \mathrm{MeV}$ $\quad f_\pi = 92.4\ \mathrm{MeV}$

$-\langle\bar{\psi}\psi\rangle^{1/3} = 250.8\ \mathrm{MeV}$ $\quad m_q^0 = 300\ \mathrm{MeV}$

$$\mathrm{m}_\pi^2 = -\frac{m_c\langle\bar{\psi}\psi\rangle}{f_\pi^2}$$

The three flavor NJL effective model for quarks

$$\mathcal{L}_f = \bar{\psi}_f \left[\gamma_\mu \left(i\partial^\mu - q_f A^\mu\right) - \hat{m}_c\right] \psi_f \; + \; \mathcal{L}_s \; + \; \mathcal{L}_d$$

where

$$\mathcal{L}_s \; = \; G \sum_{a=0}^{8} \left[(\bar{\psi}_f \lambda_a \psi_f)^2 + (\bar{\psi}_f i\gamma_5 \lambda_a \psi_f)^2\right]$$

$$\mathcal{L}_d \; = \; -K \left\{\det_f \left[\bar{\psi}_f (1+\gamma_5)\psi_f\right] + \det_f \left[\bar{\psi}_f (1-\gamma_5)\psi_f\right]\right\}$$

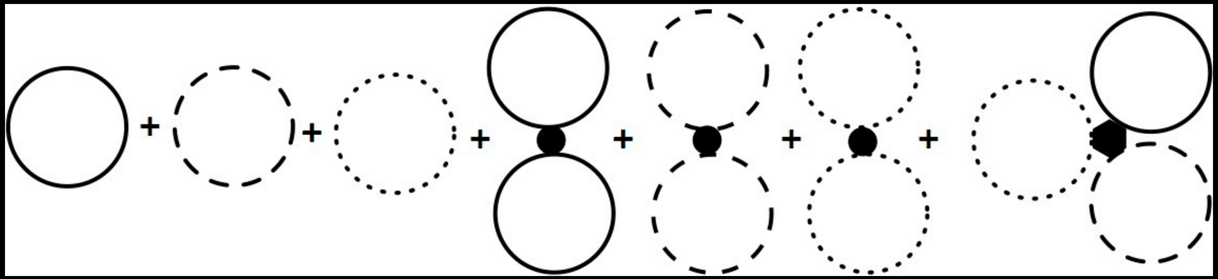
$\psi_f = (u, d, s)^T$ represents a quark field with three flavors, $\hat{m}_c = \mathrm{diag}_f(m_u, m_d, m_s)$

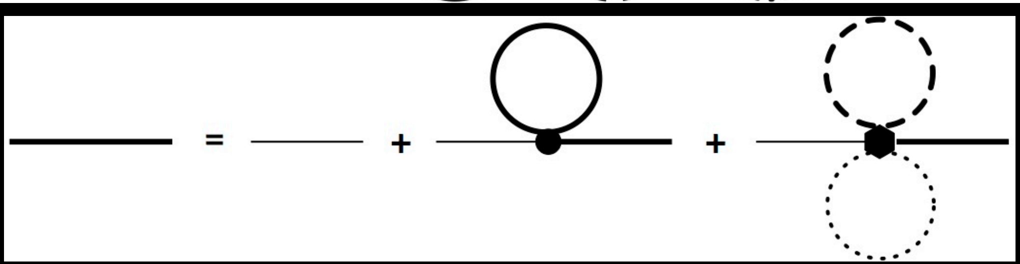
Parameters as two flavor case plus:

$m_s = 135.7\mathrm{MeV}$

$K\Lambda^5 = 9.29$

The NJL PRESSURE in the MFA (aka large N_c)





$$P_f = \theta_u + \theta_d + \theta_s - 2G(\phi_u^2 + \phi_d^2 + \phi_s^2) + 4K\phi_u\phi_d\phi_s$$

$$\theta_f = -\frac{i}{2}\mathrm{tr}\int \frac{d^4p}{(2\pi)^4} \ln\left(-p^2 + M_f^2\right)$$

$$\phi_f = \langle\bar{\psi}_f\psi_f\rangle = -i\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}\frac{1}{(\not{p} - M_f + i\epsilon)}$$

(self consistent) Effective Mass: $$M_i = m_i - 4G\phi_i + 2K\phi_j\phi_k$$

MODIFIED FEYNMAN RULES to ACCOUNT FOR: **T , m and B.**

$$p_0 \to i(\omega_\nu - i\mu_f) \ ,$$

$$\mathbf{p}^2 \to p_z^2 + (2n+1-s) \quad \text{, with} \quad s = \pm 1 \quad , n = 0, 1, \ldots$$

$$\int \frac{d^4p}{(2\pi)^4} \to i\frac{T|q_f|B}{2\pi} \sum_{\nu=-\infty}^{\infty} \sum_{n=0}^{\infty} \int \frac{dp_z}{(2\pi)} \ .$$

DIMENSIONAL REDUCTION: 4d → 1d

QM: motion in the x-y plane quantized in units of 2qB due to field along z.
Levels for which the values of p_x^2+p_y^2 lie between 2qnB and 2qB(n+1) coalesce in single level characterized by n. Must sum over n: Landau Level

After that we get the explicit relations :

$$\theta_f^{vac} = -\frac{N_c}{8\pi^2}\left\{M_f^4 \ln\left[\frac{(\Lambda+\epsilon_\Lambda)}{M_f}\right] - \epsilon_\Lambda \Lambda\left(\Lambda^2+\epsilon_\Lambda^2\right)\right\}$$

where we have defined $\epsilon_\Lambda = \sqrt{\Lambda^2 + M_f^2}$ with Λ

$$\theta_f^{mag} = \frac{N_c(|q_f|B)^2}{2\pi^2}\left[\zeta'(-1,x_f) - \frac{1}{2}(x_f^2 - x_f)\ln x_f + \frac{x_f^2}{4}\right]$$

where $x_f = M_f^2/(2|q_f|B)$ while $\zeta'(-1,x_f) = d\zeta(z,x_f)/dz$

$$\theta_f^{med} = \frac{N_c}{2\pi}\sum_{s,n,f}(|q_f|B)\int\frac{dp_z}{(2\pi)} \times \left\{T\ln[1+e^{-[E_p(B)+\mu_f]/T}] + (\mu_f \to -\mu_f)\right\}$$

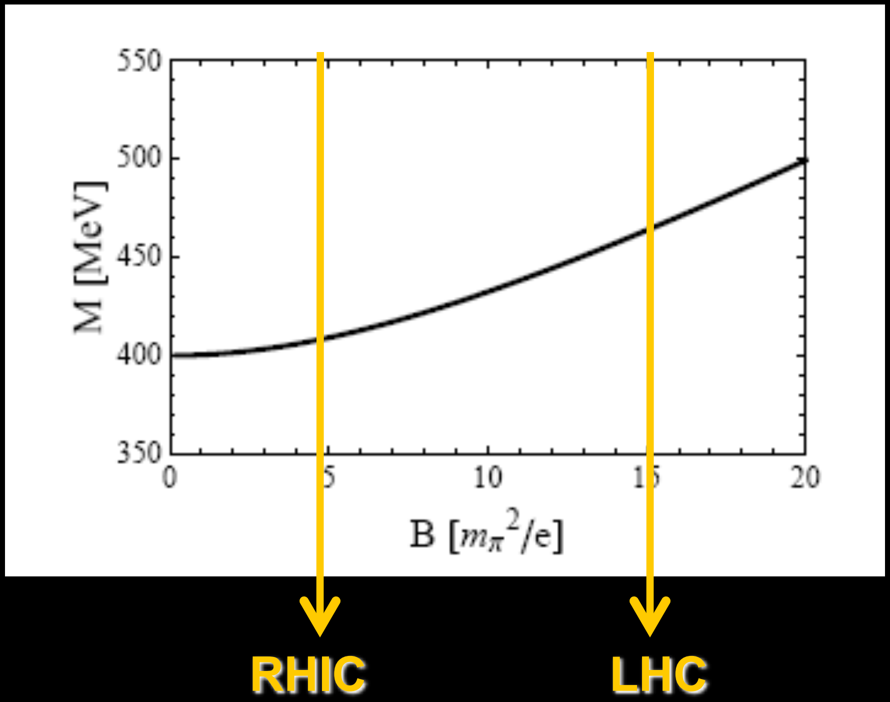
where $E_p(B) = \sqrt{p_z^2 + (2n+1-s)|q_f|B + M^2}$.

[redacted]

Also: $\phi_f \sim d\theta_f/dM_f$

We're all set now !!

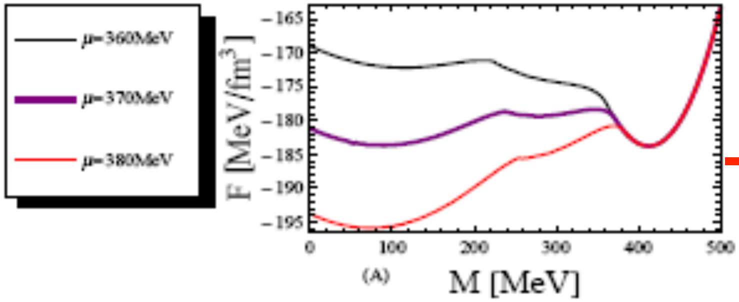
MAGNETIC CATALYSIS at T = 0 and μ = 0



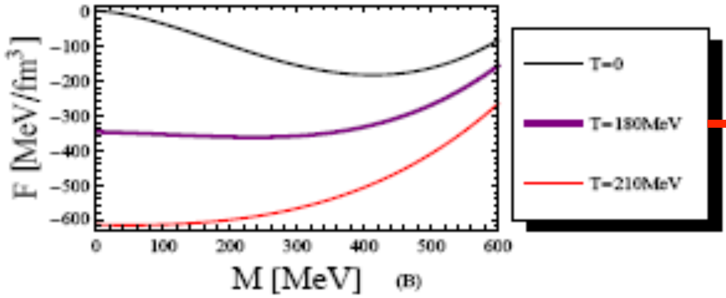
<u>QCD</u> : B stabilizes vacuum chiral asymmetry favoring qbar-q pair formation in opposition to:

<u>BCS</u> : B alignes electronic spins breaking ee pairs

TWO FLAVOR

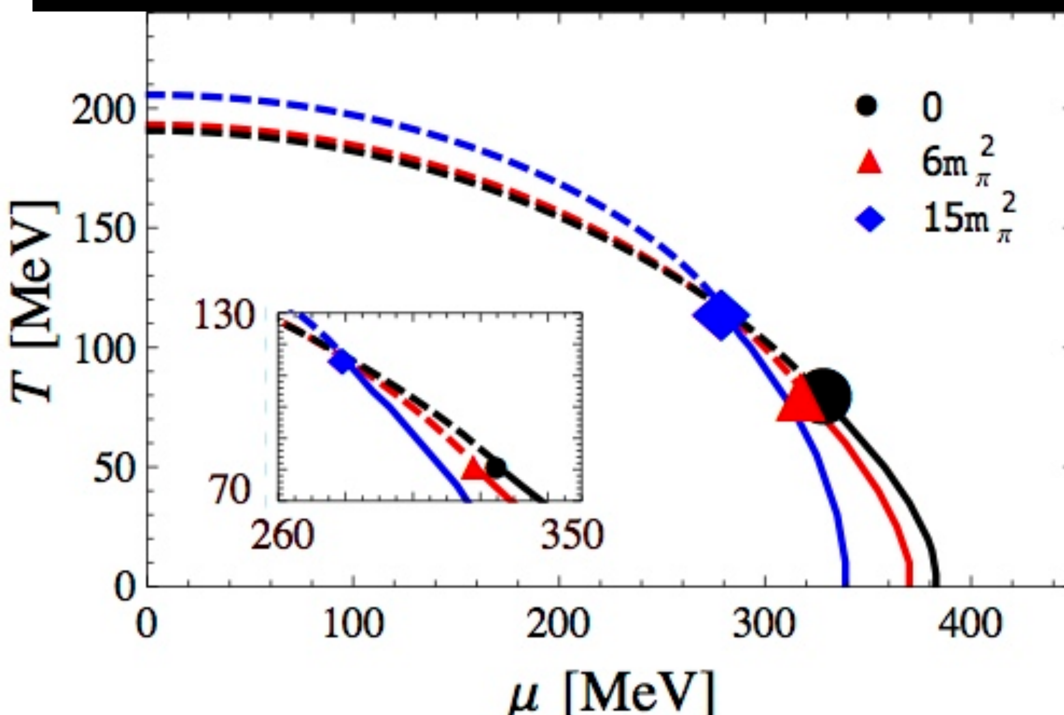


T = 0 , 1st order & oscillations LL



μ = 0, Xover

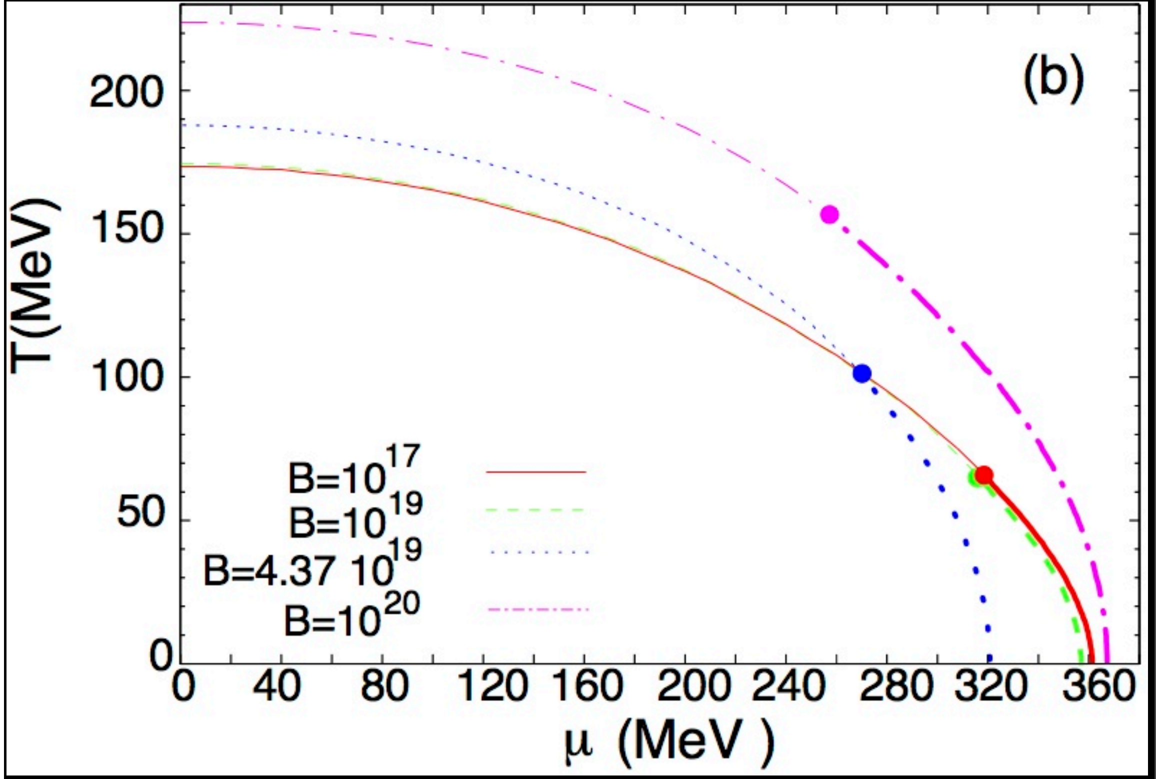
- Tpc ***increases*** at μ = 0
- Tc decreases at high μ
- CP at higher T and lower μ
- Size of 1st order line increases with B
- IMC at low T



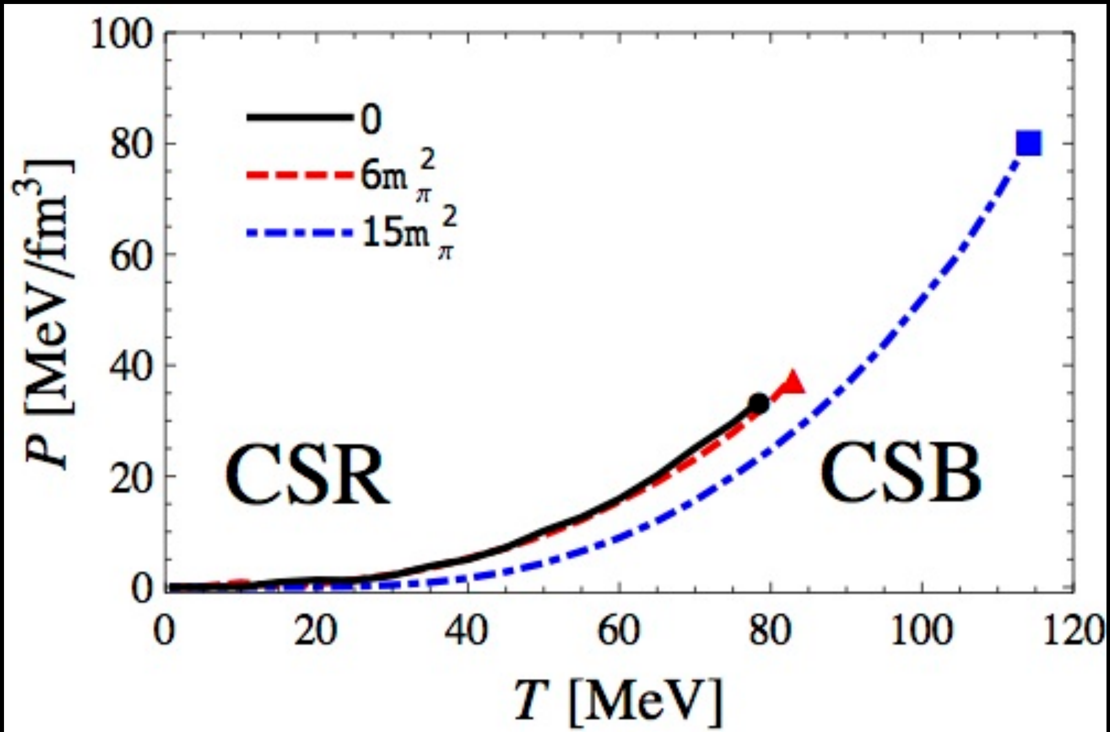
THREE FLAVOR:
Strangeness is important for the QCD-PD & for Astrophysics

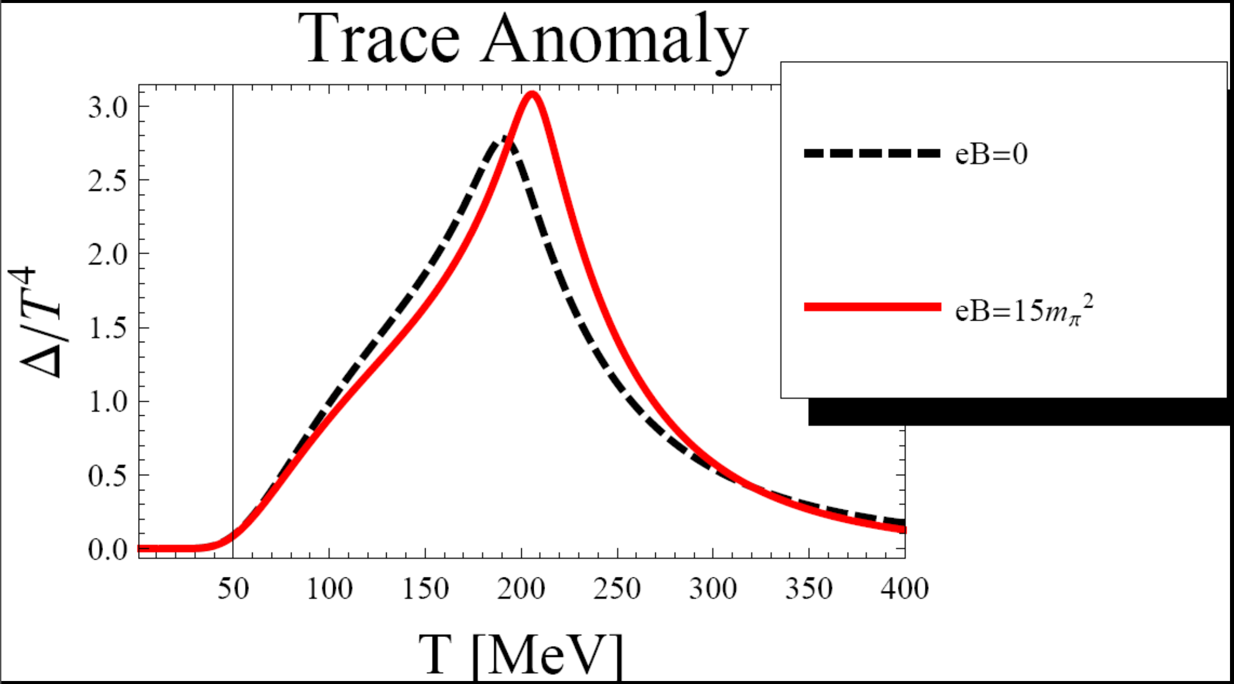
As 2 flavor case:

- Tpc ***increases*** at μ = 0
- Tc decreases at high μ
- CP at higher T and lower μ
- Size of 1st order line increases with B
- Also 1st order oscillates for very high fields
- IMC at low T



OTHER QUANTITIES





$$\Delta = \frac{\epsilon - 3P}{T^4}$$

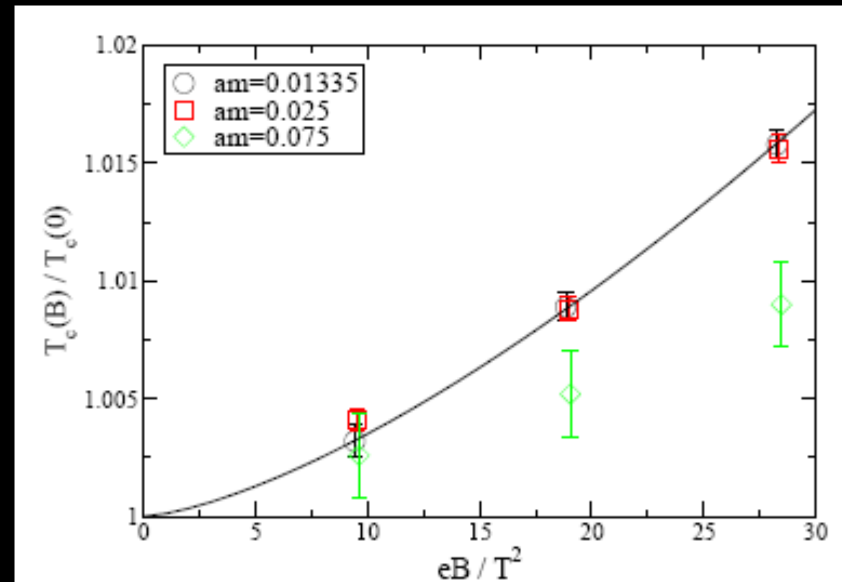
μ=0

LATTICE RESULTS at μ=0 and High T

M. D'Elia et al. (2010)

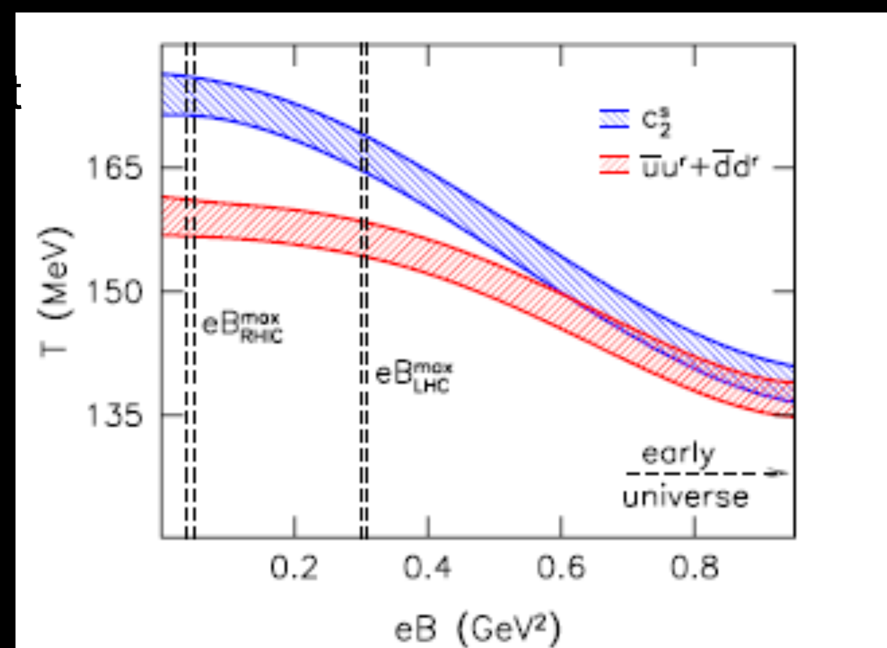
Nf=2, large *a* & high π mass

As Model Predictions BUT...



???

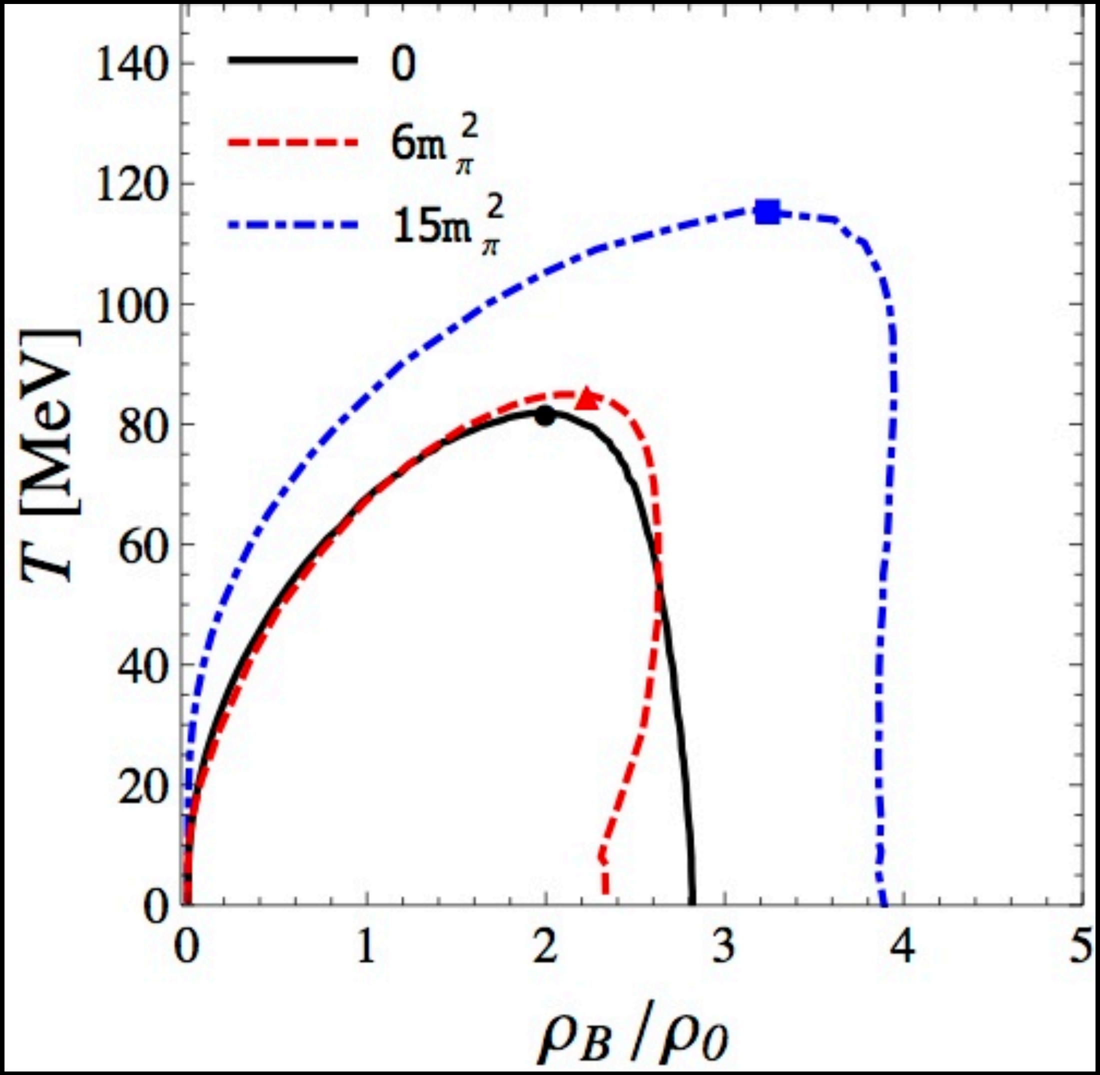
Fodor's Group (2011)



Nf=2+1, extrapolation to continuum

& physical π mass

LOW TEMPERATURE - HIGH DENSITY



Coexistence Region:

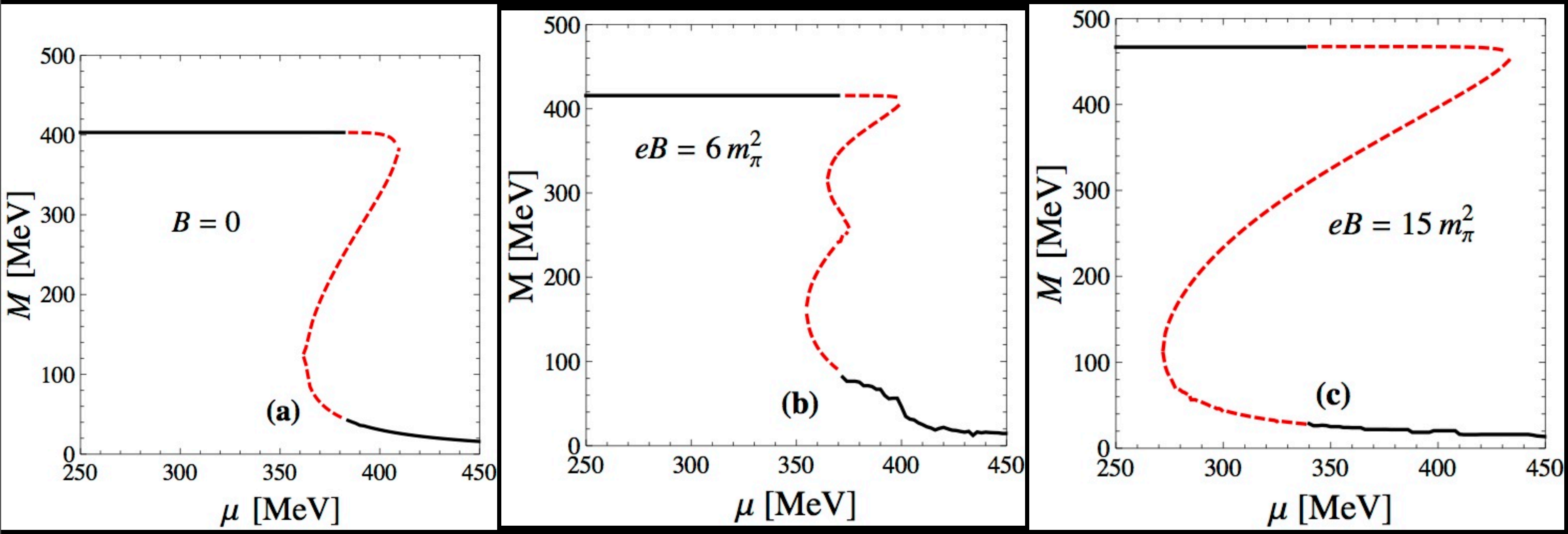
low ρ = 0

high ρ OSCILLATES

around B=0 value !!

WHY??

EFFECTIVE QUARK MASS AT T = 0

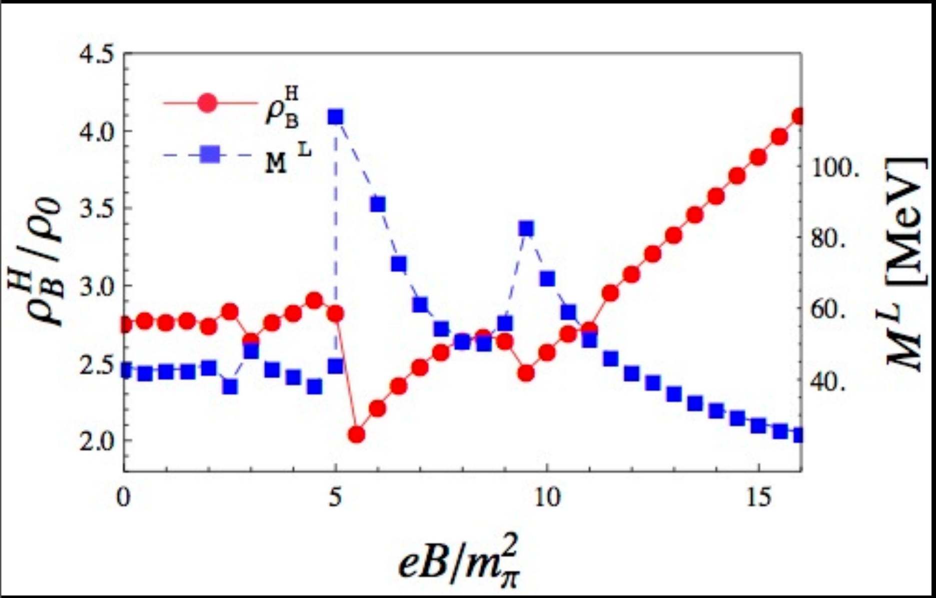


$$\rho_B(\mu, B) = \theta(k_F^2) \sum_{f=u}^{d} \sum_{k=0}^{k_{f,max}} \alpha_k \frac{|q_f| B N_c}{6\pi^2} k_F$$

where $k_F = \sqrt{\mu^2 - 2|q_f|kB - M^2}$ and

$$k_{f,max} = \frac{\mu^2 - M^2}{2|q_f|B}$$

de Haas - van Alphen Oscillations at T=0

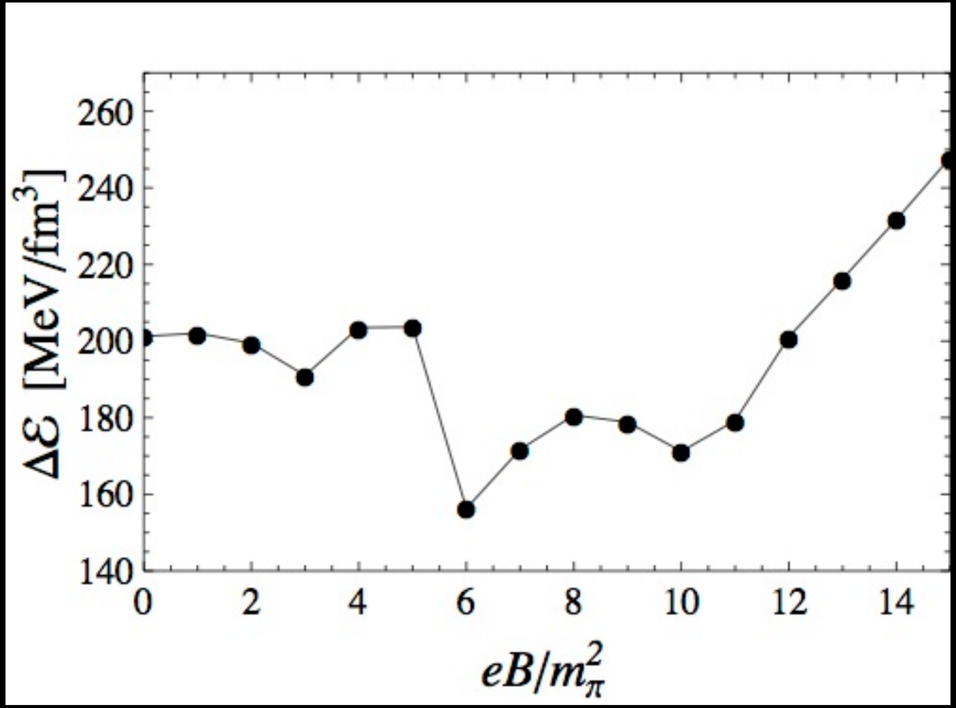


Peak at eB ~ 5.5 $k_{u,max}$ 1 to 0

Peak at eB ~ 9.5 $k_{d,max}$ 1 to 0

For eB > 9.5 only LLL

Latent Heat:



1st low T application: the SURFACE TENSION

$$\gamma_T = a \int_{\rho_L}^{\rho_H} \frac{d\rho}{\rho_g} [2\mathcal{E}_g \Delta f_T(\rho)]^{1/2}$$

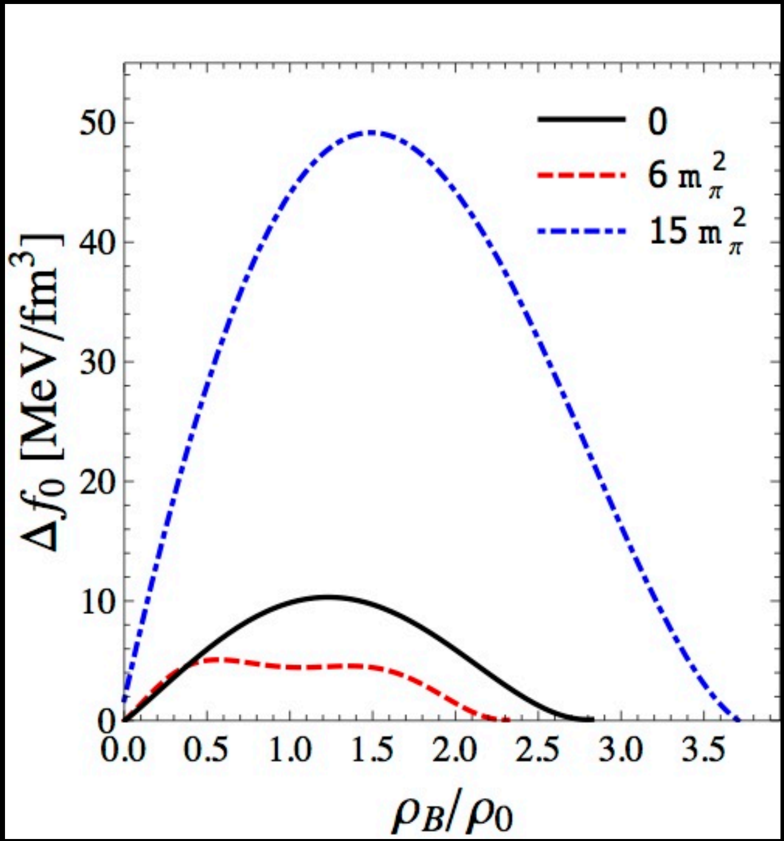
(J. Randrup 2009)

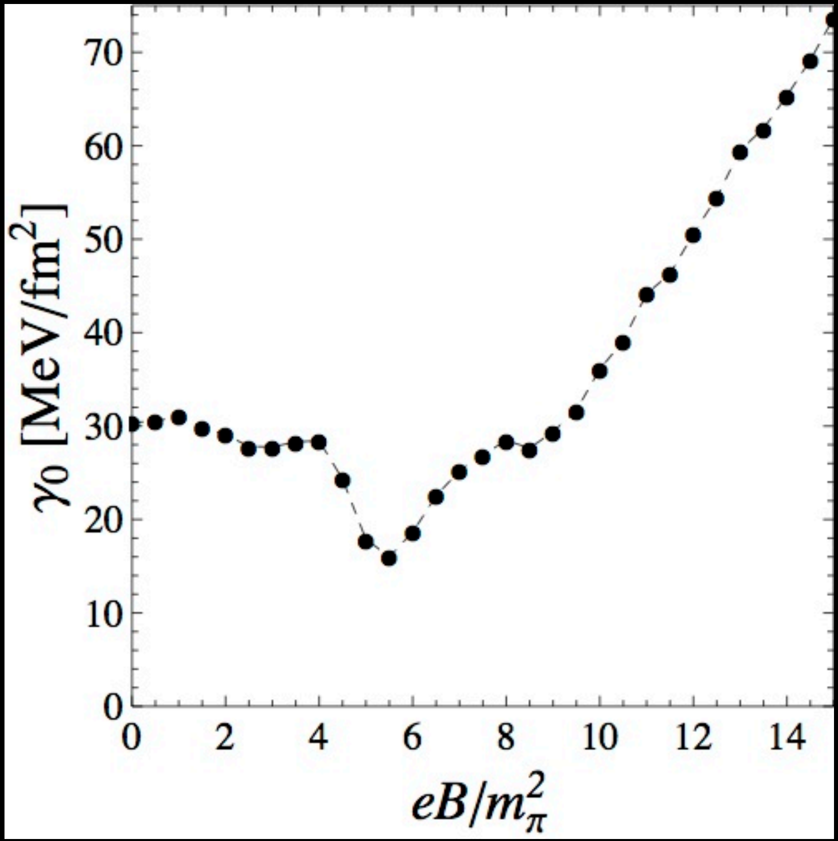
$\Delta f_T(\rho) \equiv f_T(\rho) - f_T^M(\rho) \geq 0$: $f_T(\rho)$ and $f_T^M(\rho)$ coincide at the two coexistence densities and $f_T(\rho) = -P(\rho) + \rho\mu(\rho)$ exceeds $f_T^M(\rho)$ for intermediate densities.

$\rho_g = \rho_c$ and $\mathcal{E}_g = (1/2)[\mathcal{E}_0(\rho_c) + \mathcal{E}_c]$

$a = 1/m_\sigma \approx 0.33$ fm

Surface Tension (γ): key role for the possiblity of quark star formation and existence of mixed phase in hybrid stars.
If $\gamma \lesssim 40$ MeV/ fm^2: phase conversion and mixed phase OK!

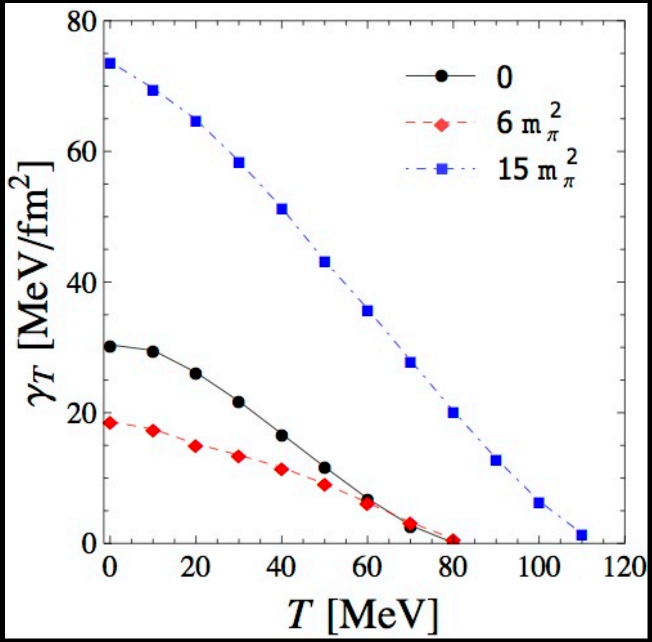




Results with NJL su2 at T=0: up to eB ≈ 10 m_π^2 ≈ 10^{19} G
$\gamma \lesssim 40$ MeV/ fm^2 !!! A.F. Garcia & MBP, PRC (2013)

Other effects? E.g: 1) Temperature, 2) vector interaction, 3) strangeness , confinement (Polyakov-NJL) etc

1) Temperature:



2) **vector interaction**: weakens 1st order transition => should further decrease γ

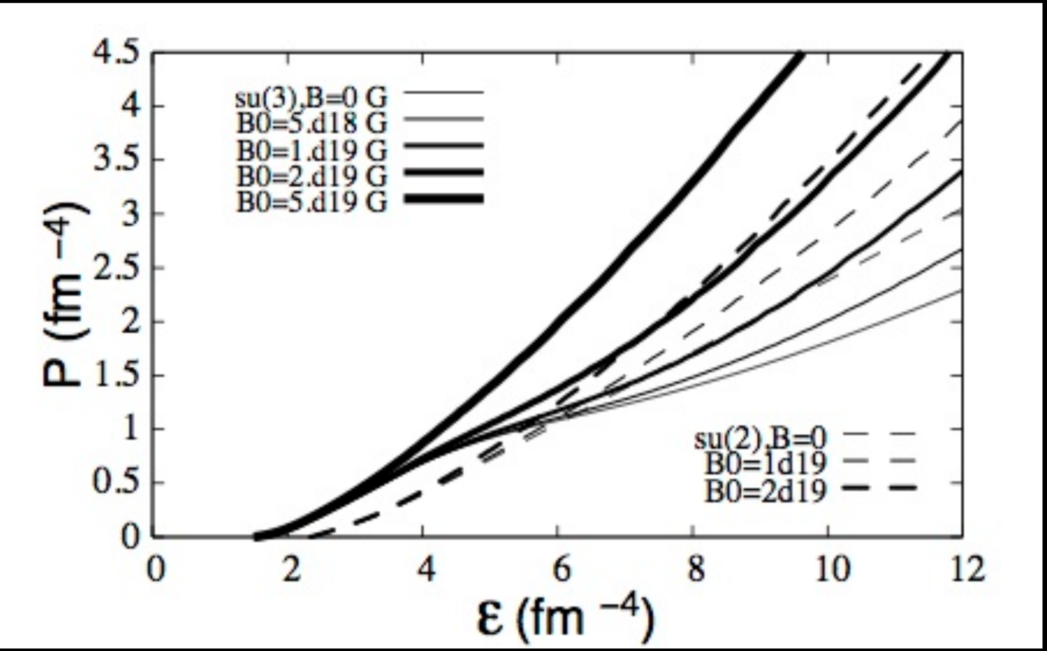
3) **strangeness : negligible effect (MBP, Koch & Randrup)**

4) **Polyakov: negligible effect (Mintz, Ramos et al.)**

2,3 & 4 are results obtained at B=0.

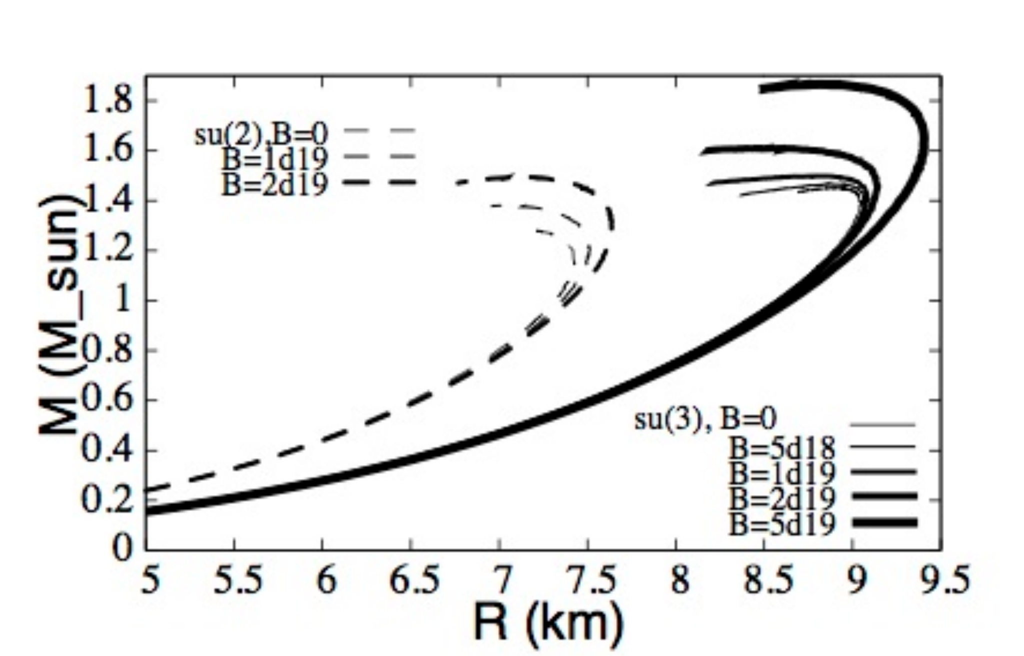
2nd low T application: EoS & TOV with NJL su3

EoS becomes harder with increasing B



Higher stellar masses with increasing B.

S. Avancini, D.P. Menezes, MBP & C. Providencia PRC80, 065805(2009)



CONCLUSIONS

- At $\mu=0$ Tc INCREASES with B as in most models but contrary to recent by Bali, Fodor et al (2011). Open question at high T....
- For moderate μ, Tc DECREASES with B..
- "Size" of 1st order line INCREASES with B and so does the coexistence region: astrophysical consequences for quark star formation??
- At low T: coexistence μ OSCILLATES around the B=0 value. Critical chemical potential value first decreases (IMC) then increases for very high B (but higher than cut off...).
- 1st low T application: the surface tension still low enough (and even decreases for intermediate B) so as to allow for mixed phase.
- 2nd low T application: harder EoS and higher masses for quark stars

Collaborators:

Graduate students UFSC: Gabriel Ferrari, Robson Denke & Andre Garcia.

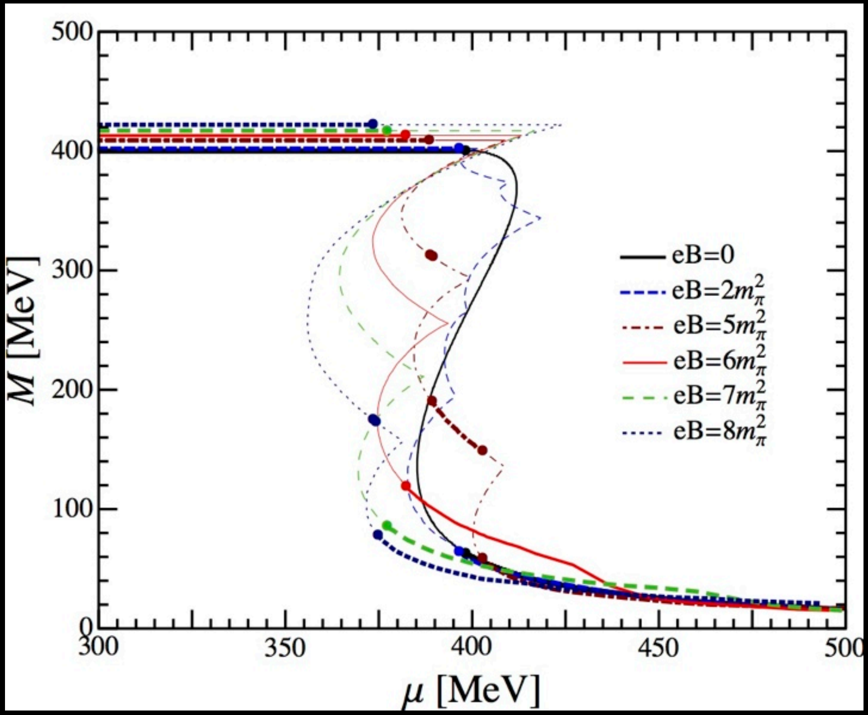
Faculty UFSC: Sidney Avancini & Débora P. Menezes

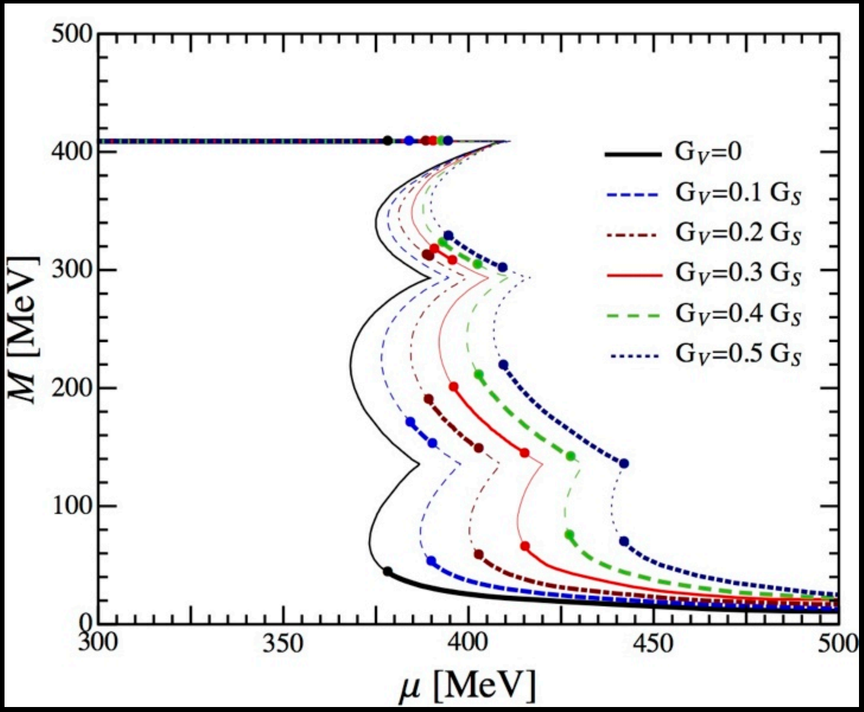
External: Constança Providência (Portugal)

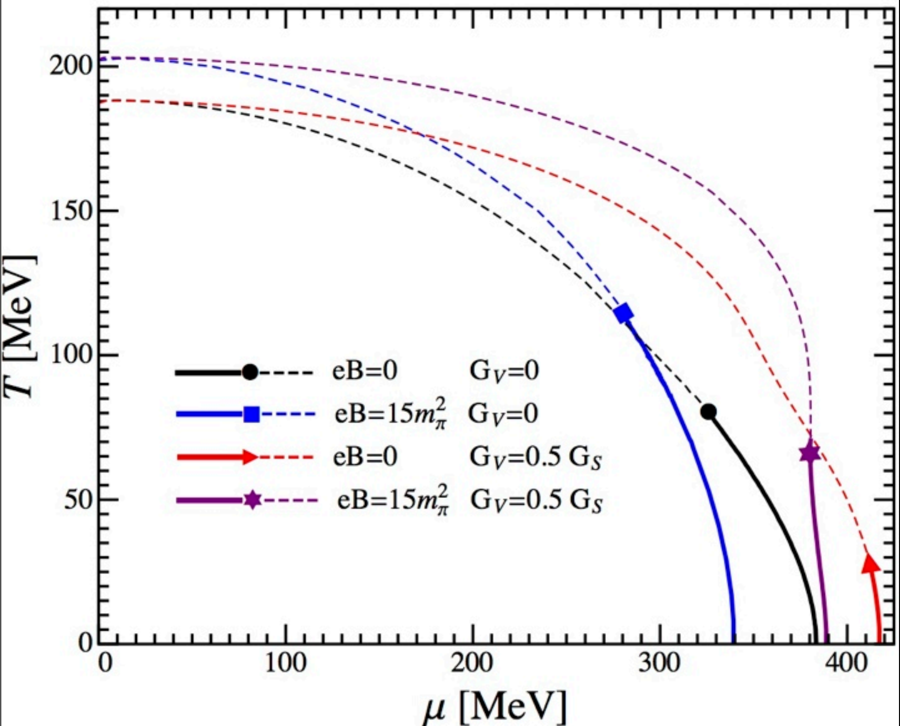
SPONSORS:

3rd Application: Repulsive Vector Channel

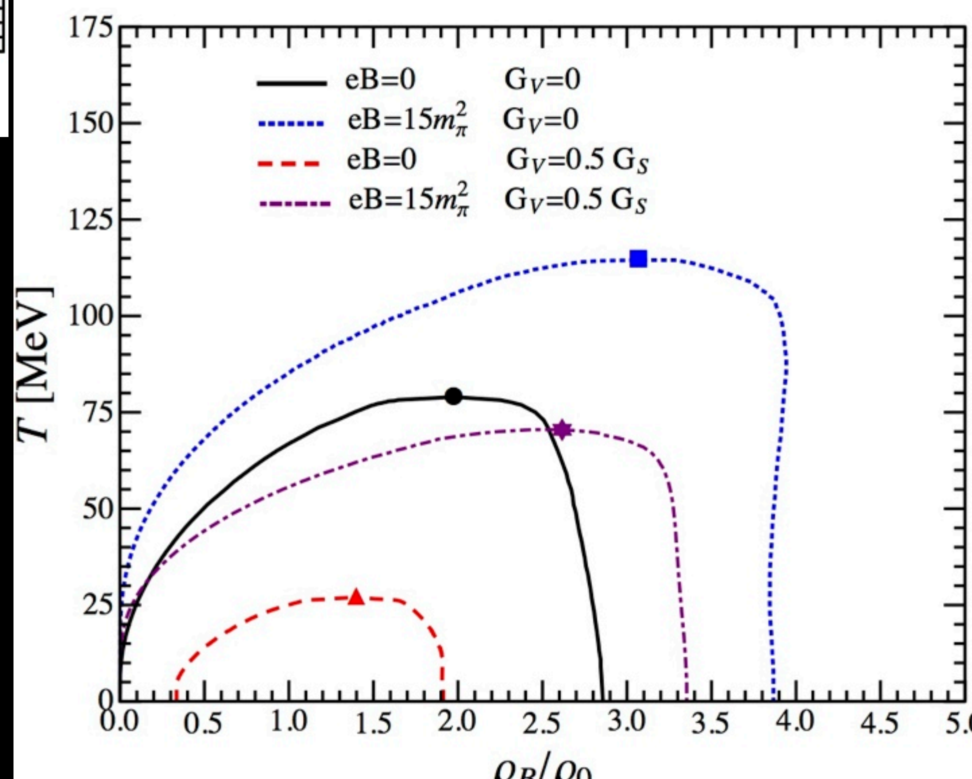
$$\text{addterm}: \quad -G_V(\bar{\psi}\gamma_\mu\psi)^2$$





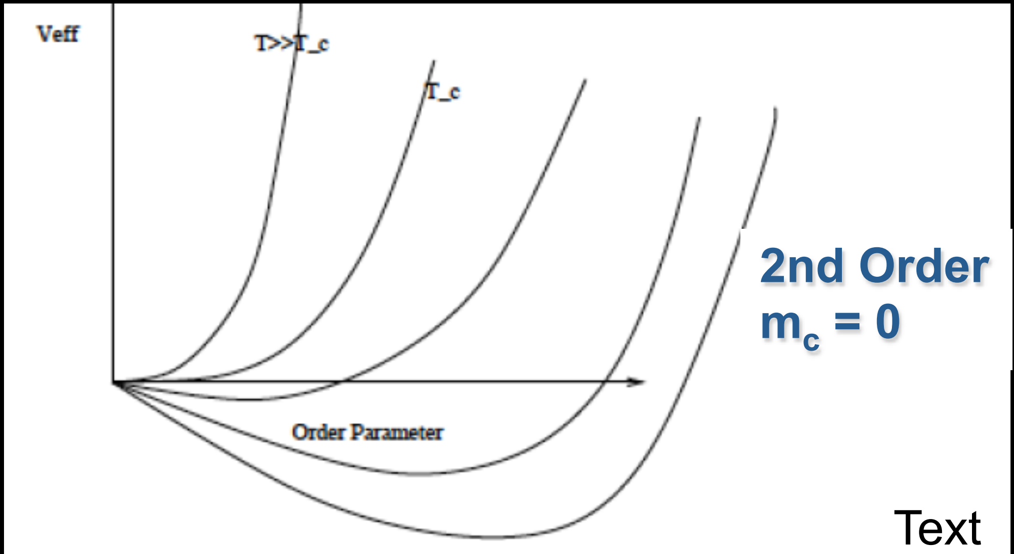


T [MeV]
μ [MeV]
eB=0 G_V=0
eB=15m_π^2 G_V=0
eB=0 G_V=0.5 G_S
eB=15m_π^2 G_V=0.5 G_S



T [MeV]
ρ_B/ρ_0
eB=0 G_V=0
eB=15m_π^2 G_V=0
eB=0 G_V=0.5 G_S
eB=15m_π^2 G_V=0.5 G_S

2nd ORDER PHASE TRANSITION & X-OVER



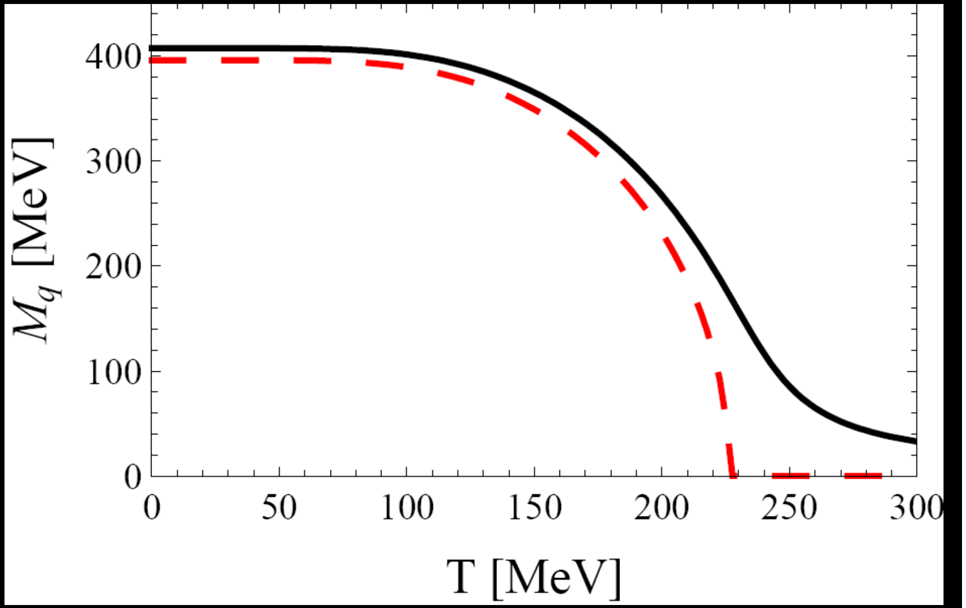
2nd Order
$m_c = 0$

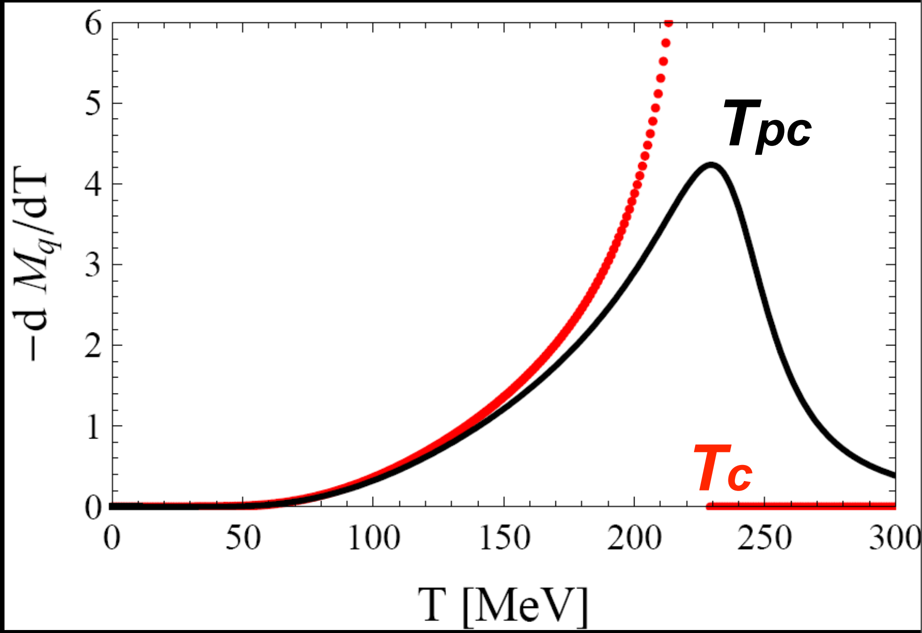
REMEMBER:

$$m_\pi^2 = -\frac{m_c\langle\bar{\psi}\psi\rangle}{f_\pi^2}$$

REAL WORLD: $m_c \neq 0$

Chiral Symmetry
NOT EXACT !!!



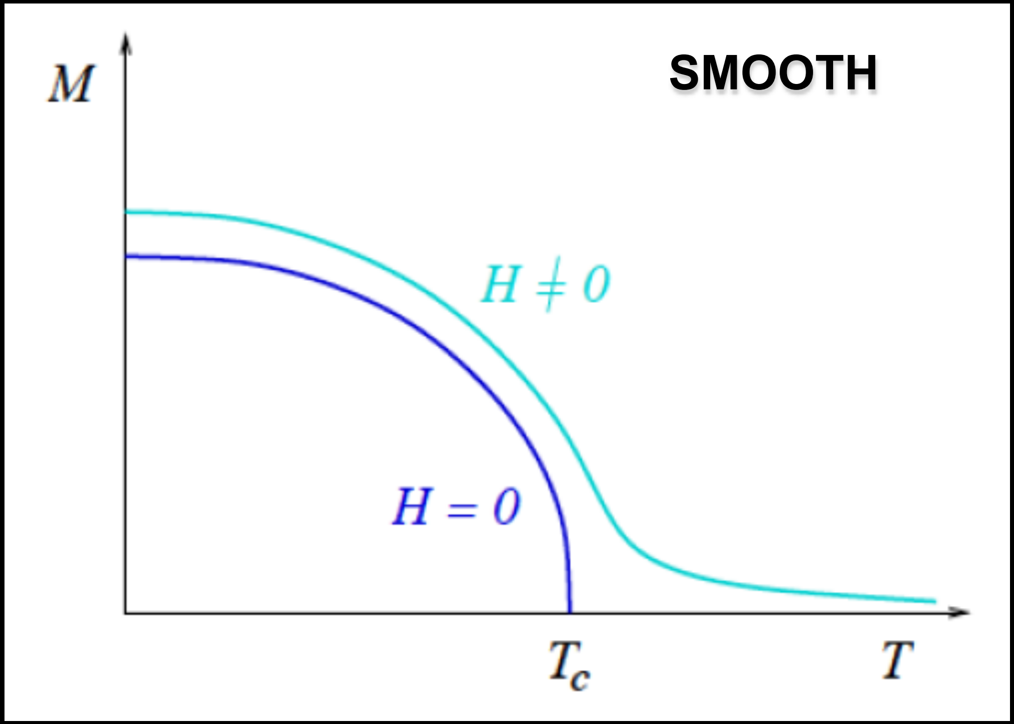


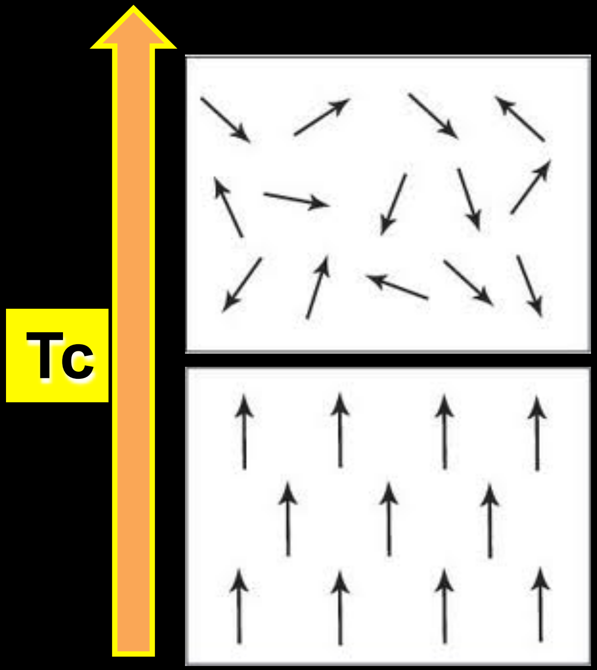
In summary, the situation is similar to the one found in a ferromagnet:

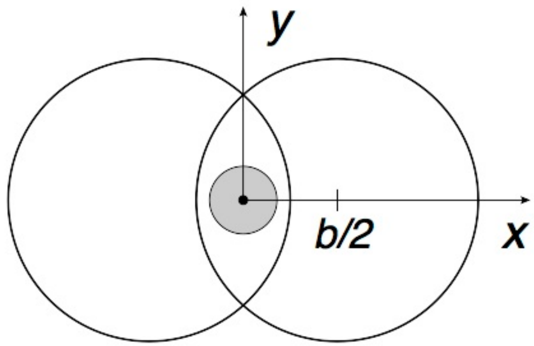
Magnetization analogue to quark condensate (which leads to quark effective mass)

Background magnetic field analogue to current quark mass:

switches 2nd order to X-over



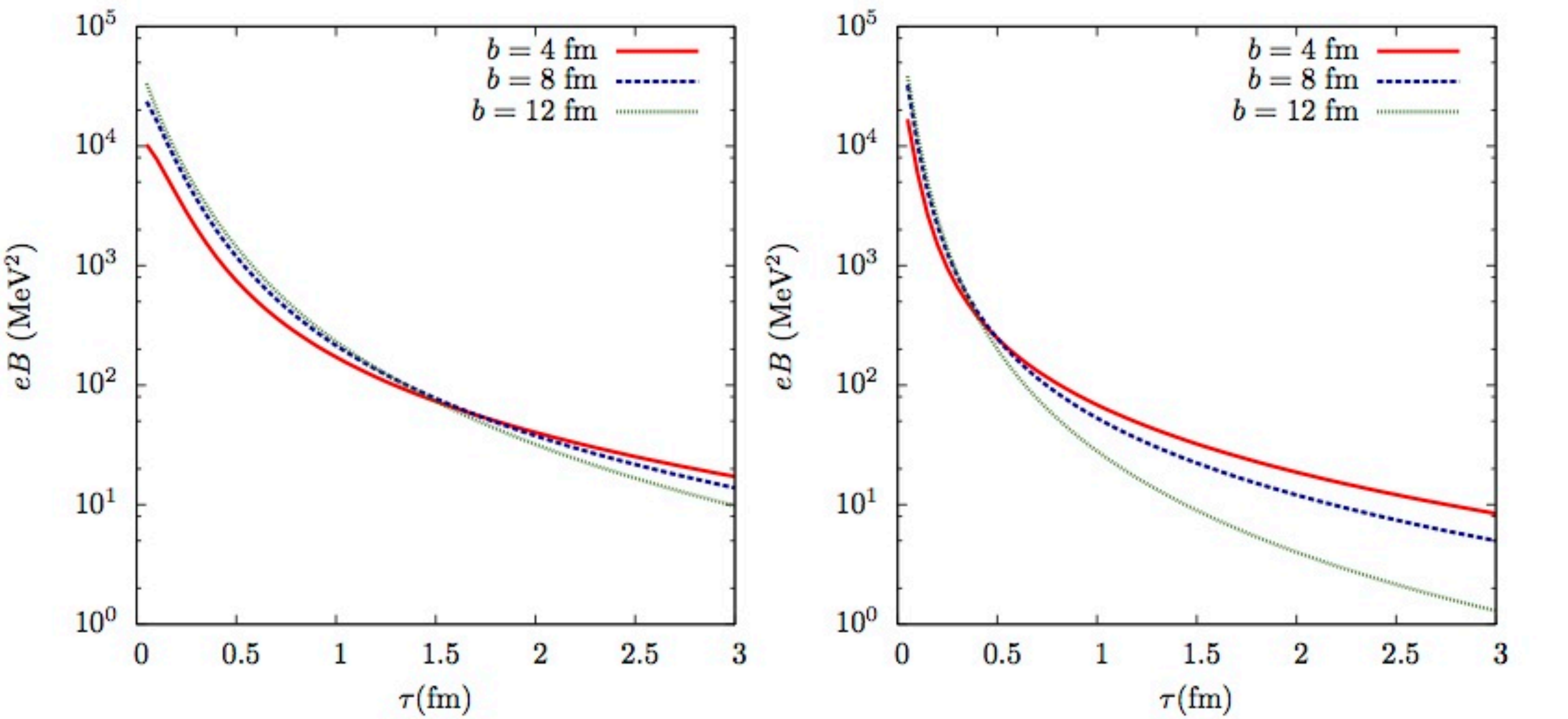




10^5 MeV^2 ~ 5 m_π^2 ~ 5 x 10^{18} G

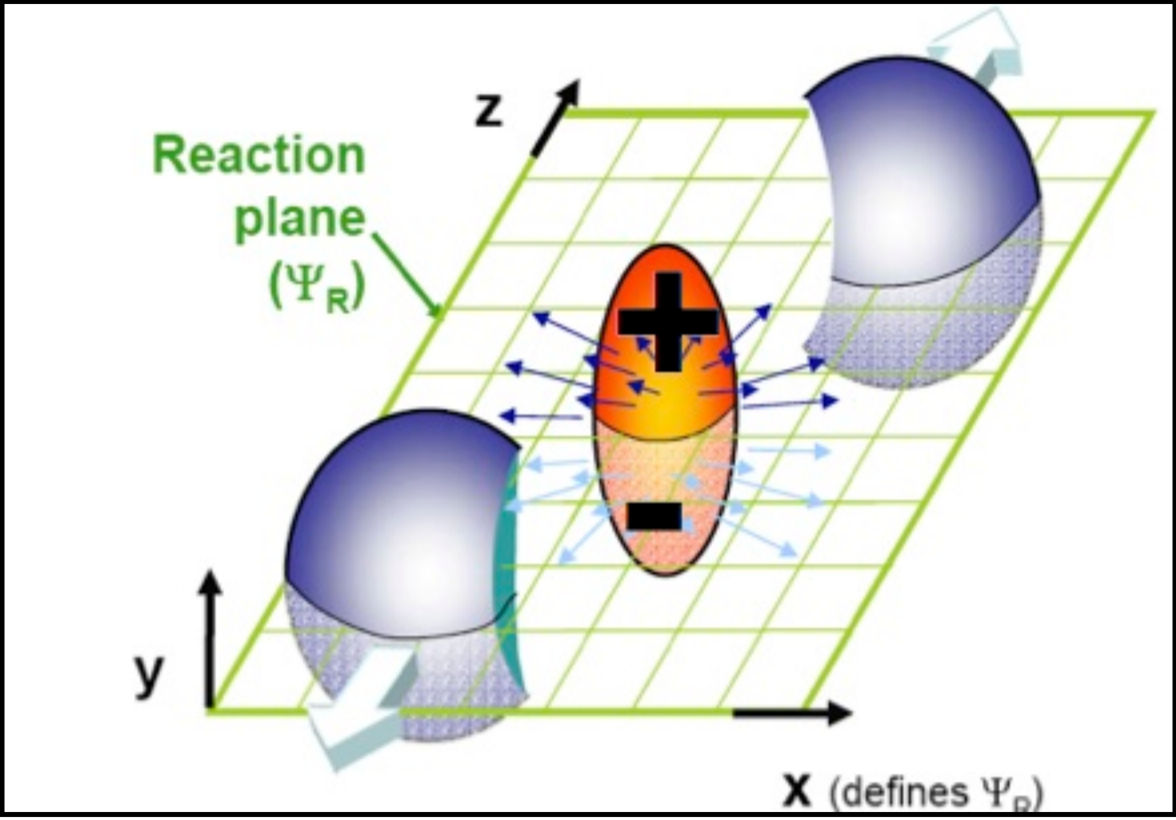
formation QGP few tenth fm/c

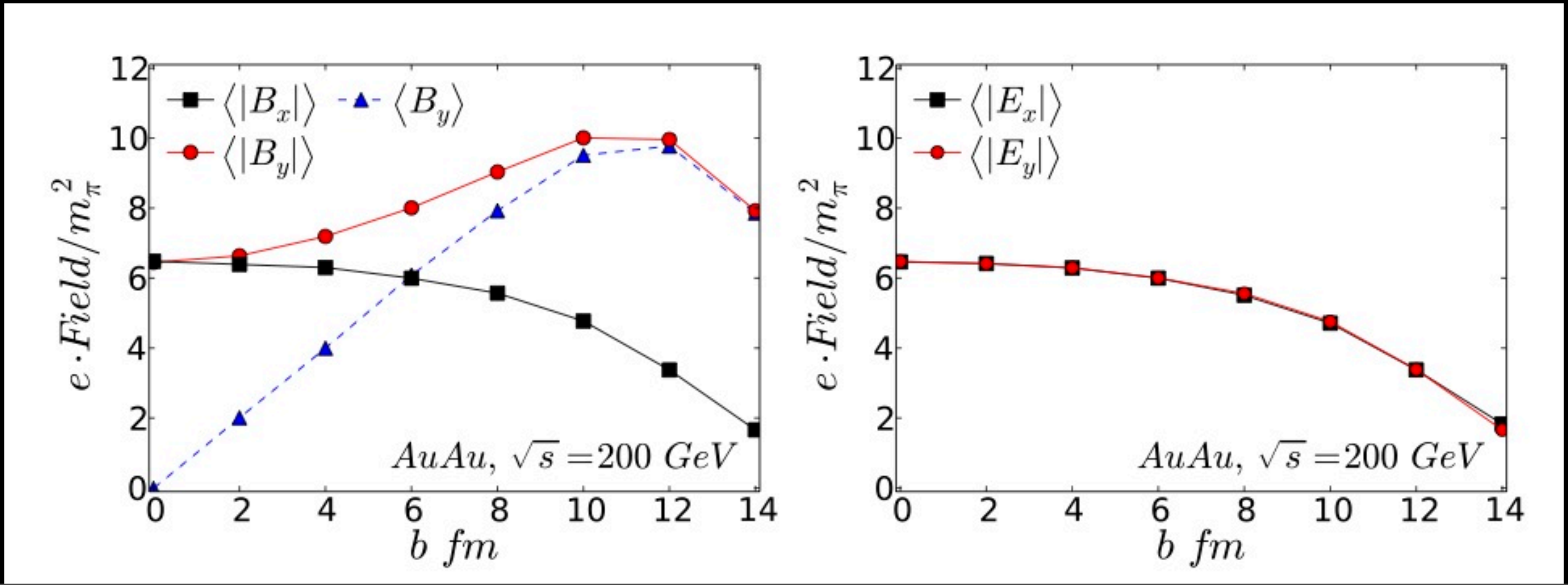
B relaxation time 1-2 fm/c



RHIC@BNL

$$eB(\tau = 0.2\,\mathrm{fm}) = 10^3 \sim 10^4\ \mathrm{MeV}^2 \sim 10^{17}\,\mathrm{G}$$


z
Reaction plane (Ψ_R)
y
x (defines Ψ_R)


$\langle |B_x| \rangle$
$\langle B_y \rangle$
$\langle |B_y| \rangle$
$e \cdot Field/m_\pi^2$
$AuAu,\ \sqrt{s} = 200\ GeV$
$b\ fm$
$\langle |E_x| \rangle$
$\langle |E_y| \rangle$
$e \cdot Field/m_\pi^2$
$AuAu,\ \sqrt{s} = 200\ GeV$
$b\ fm$

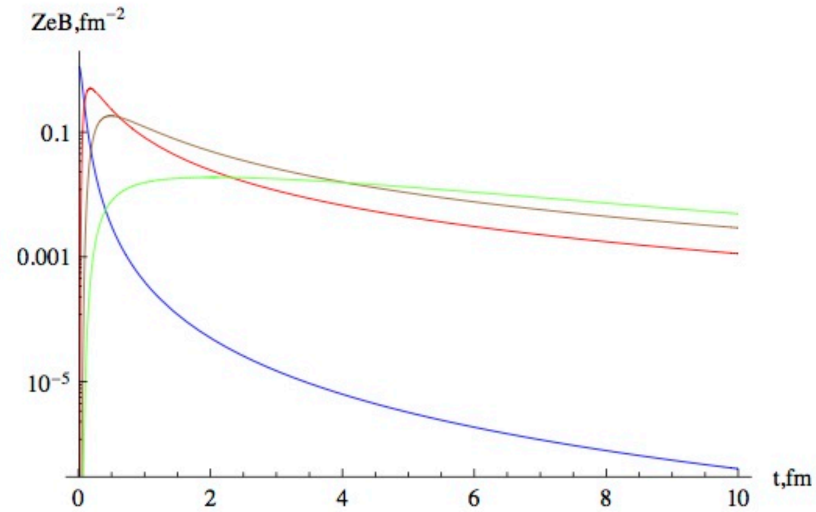


FIG. 4: Relaxation of magnetic field at $z = 0$ in vacuum (blue), in static conducting medium at $\sigma = 5.8$ MeV (red) and at $\sigma = 16$ MeV (brown) and in the expanding medium (green). Units of B is fm$^{-2} \approx 2m_\pi^2$. $b = 7$ fm, $Z = 79$ (Gold nucleus), $\gamma = 100$ (RHIC).

IMC: to form <qbar-q> costs B mu^2 at T=0

MC: infrared effects by going from 4 to 2 dim