Constraints on the Symmetry Energy

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32nd International Workshop on Nuclear Dynamics and Thermodynamics

August 19-22, 2013, College Station, Texas
Inspiration

At a Gordon Conference:

Joe: What about isospin dependence of limiting $T$?

PD: Can look at that - explored $T_{\text{lim}}$ around Diploma Thesis.

Back home: Pressure, Coulomb, surface tension. . . Ugh, surface tension can depend on $T$ and isospin as well. . .

$\Rightarrow$ Let’s explore the simplified problem of dependence of the surface tension on isospin asymmetry

PD Surface Symmetry Energy NPA727(03)233
PD&Lee NPA818(09)819; arXiv:1307.4130
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Symmetry Energy in Nuclear Mass Formula

Textbook Bethe-Weizsäcker formula:

\[ E = -a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_a \frac{(N - Z)^2}{A} + E_{\text{mic}} \]

Symmetry energy: charge \( n \leftrightarrow p \) symmetry of interactions

Analogy with capacitor:

\[ E_a = a_a \frac{(N - Z)^2}{A} \equiv \frac{(N - Z)^2}{A/a_a} \leftrightarrow E = \frac{Q^2}{2C} \]

?Volume Capacitance?

\[ E_a = \frac{(N - Z)^2}{A/a_a} \rightarrow \frac{A}{a_a} + \frac{A^{2/3}}{a_a^S} \]

Thomas-Fermi (local density) approximation:

\[ 'C' = \frac{A}{a_a(A)} = \int \frac{\rho \, dr}{S(\rho)} = \frac{A}{a_a V}, \text{ for } S(\rho) \equiv a_a^V \]

TF breaks in nuclear surface at \( \rho < \rho_0/4 \)

PD&Lee NPA818(2009)36
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PD&Lee NPA818(2009)36
Mass and Skin Fits

Symmetry Energy:

\[ E_a = \frac{a^V_a}{A} \frac{(N - Z)^2}{1 + \frac{a^V_a}{a^S_a A^{1/3}}} \]

Skin:

\[ \Delta r_{np} = \frac{2}{3} \frac{r_{rms}}{A^{1/3}} \frac{a^s_a}{a^s_a} \left( \frac{N - Z}{A} - Coul \right) \]
Fits in $L-a^V_a$ Plane

\[ \Delta r_{np} \text{ of } ^{208}\text{Pb (fm)} \]

\[ \rho_t \text{ (fm}^{-3}\text{)} \]

Overlaps

Neutron Skin Data of Sn Isotopes

Iso. Diff. & double n/p

Lie-Wen Chen et al. PRC82(10)024321
Charge Invariance

?a_a(A)? Conclusions on sym-energy details, following E-formula fits, interrelated with conclusions on other terms in the formula: asymmetry-dependent Coulomb, Wigner & pairing + asymmetry-independent, due to \((N - Z)/A\) - A correlations along stability line [PD NPA727(03)233]!

Best would be to study the symmetry energy in isolation from the rest of E-formula! Absurd?!

Charge invariance to rescue: lowest nuclear states characterized by different isospin values \((T, T_z)\), 
\(T_z = (Z - N)/2\). Nuclear energy scalar in isospin space:

\[
\text{sym energy } \quad E_a = a_a(A) \frac{(N - Z)^2}{A} = 4 a_a(A) \frac{T_z^2}{A}
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\[
\rightarrow E_a = 4 a_a(A) \frac{T^2}{A} = 4 a_a(A) \frac{T(T + 1)}{A}
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\]
\[ a_a(A) \text{ Nucleus-by-Nucleus} \]

\[ \rightarrow E_a = 4 a_a(A) \frac{T(T+1)}{A} \]

In the ground state \( T \) takes on the lowest possible value \( T = |T_z| = |N - Z|/2 \). Through ’+1’ most of the Wigner term absorbed.

Formula generalized to the lowest state of a given \( T \) (e.g. Jänecke et al., NPA728(03)23).

Lowest state of a given \( T \): isobaric analogue state (IAS) of some neighboring nucleus ground-state.

Study of changes in the symmetry term possible nucleus by nucleus

PD&Lee arXiv:1307.4130

Symmetry Energy

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Queries in the Context of Data

Are expansions valid? Coefficient values??

\[ E_{\text{IAS}}^* = E_{\text{IAS}} - E_{\text{gs}} \approx \frac{4a_a(A)}{A} \Delta \left[ T(T+1) \right] + \Delta E_{\text{mic}} \]

Is the excitation energy linear in the isospin squared??

\[ \frac{A}{a_a(A)} \approx \frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S} \]

or

\[ a_a^{-1} \approx (a_a^V)^{-1} + (a_a^S)^{-1} A^{-1/3} \]

Is the volume-surface separation valid?

⇒ From an \( a_a^V - a_a^S \) fit can one learn about \( a_a^V \) and \( L \) for uniform matter?
Insight into IAS Analysis

IAS data: Antony et al. ADNDT66(97)1
Shell corrections: Koura et al. ProTheoPhys113(05)305

Excitation energies to IAS, $E_{\text{IAS}}^*$, for different $A$, lumped together in narrow regions of $A$. Is $E_a \propto T(T+1)$? YES
Excitation energies to IAS, $E_{\text{IAS}}^*$, for different $A$, lumped together in narrow regions of $A$. 

Is $E_a \propto T(T+1)$? YES
\[ a_a(A) = \frac{A}{4} \frac{E_{\text{IAS}}^* - \Delta E_{\text{mic}}}{\Delta T^2} \]

\( E_{\text{mic}} \) from: Koura et al.
ProTheoPhys113(05)305

Heavy nuclei \( a_a \sim 22 \text{ MeV} \), light \( a_a \sim 10 \text{ MeV} \) (more surface + low \( \rho \))
**Z-Dependence of Symmetry Coefficients?**

Symmetry coefficients on a nucleus-by-nucleus basis

- Surface less dominant, higher average $\rho$
- Surface more dominant, lower average $\rho$
Sensitivity to Shell Corrections

Fit to raw data ($A > 30$) in the middle, but:

Moller et al. fit: $a_a^V = 39.73$ MeV, $a_a^S = 8.48$ MeV
von Groote et al.: $a_a^V = 31.74$ MeV, $a_a^S = 11.27$ MeV
Comparisons to Skyrme-Hartree-Fock

Issues in data-theory comparisons (codes by P.-G. Reinhard):
1. No isospin invariance in SHF - impossible to follow the procedure for data
2. Shell corrections not feasible at such scrutiny as for data
3. Coulomb effects.

Solution: Procedure that yields the same results as the energy, in the bulk limit, but is weakly affected by shell effects:

\[
\frac{(N - Z)_{r < r_c}}{N - Z} = \frac{C_{r < r_c}}{C} = \frac{a_a}{A a_a^V} \int_{r < r_c} \frac{\rho}{S(\rho)}
\]
$a_a(A)$ from Mean-Field Calculations

Skyrme-Hartree-Fock theory (codes by P.-G. Reinhard)

Similar behavior with $A$ as for IAS

IAS (raw) vs ska25s20

ska25s20

$a_a^V = 33.8$ MeV
$L = 63.8$ MeV
$a_a(A)$ from Different Mean Fields

- **Skyrme-Hartree-Fock**
  - SkM1
    - $a^v_a = 25.2$ MeV
    - $L = -35.3$ MeV
- **IAS Analysis**
  - $a^v_a = 33.8$ MeV
  - $L = 63.8$ MeV

**Questions**: Slope $L$ in $\rho$ \iff slope in $A$?

Less impact of the slope $L$ at $\rho_0$ than expected!

??Difficulty for $L$ determination??
**Model-Independent Large-A Expansion??**

Symbols: results of spherical no-Coulomb SHF calcs

⇒ Lines: volume-surface decomposition - expectation vs fit

→ Symmetric matter energy f/sample Skyrmes

∼ Works

→ Symmetry coefficient

∼ Not...

Expectations from half-∞ matter.
Can $S(\rho)$ Be Constrained?! 

Pearson correlation coefficient 

$$ r_{XY} = \frac{\langle (X - \langle X \rangle)(Y - \langle Y \rangle) \rangle}{\sqrt{\langle (X - \langle X \rangle)^2 \rangle \langle (Y - \langle Y \rangle)^2 \rangle}} $$

$|r| \sim 1$ - strong correlation

$r \sim 0$ - no correlation

$X \equiv a_a(A)$

$Y \equiv S(\rho)$

Ensemble of Skyrmes

Nearly no information about $S(\rho_0)$!
Symmetry-Energy Correlations When Strong

uniform matter

A = 240
ρ = 0.105 fm$^{-3}$

Vehicle: phenomenological mean-field theory

NO $S(\rho) \approx a_a$!
Constraints on Symmetry Energy $S(\rho)$

Demand that Skyrme approximates IAS results at $A > 30$ produces a constraint area for $S(\rho)$:

\[ S(\rho) \]

[Slope constraint??]
Asymmetry Skin & Energy Stiffness

Pearson coef of
\[ \Delta r_{np} = r_{np}^{\text{rms}} - r_{\rho}^{\text{rms}} \]
& stiffness of \[ S \]
\[ \gamma(\rho) := \frac{\rho}{S} \frac{dS}{d\rho} \]
f/different \( A \) at fixed \( \eta = (N - Z)/A \)
Asymmetry Skin & Energy Stiffness

Pearson coef of \( \Delta r_{np} \) & stiffness

\[
\gamma(\rho) := \frac{\rho}{S} \frac{d S}{d \rho}
\]

f/different

\( \eta = (N - Z)/A \)

at fixed \( A \)
# Asymmetry Skins from Measurements

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Reference</th>
<th>Data Source</th>
<th>$\Delta r_{np}$ [fm]</th>
<th>$\Delta r_{np}^{GF}$ [fm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}$Ca</td>
<td>Friedman [92]</td>
<td>pionic atoms</td>
<td>0.13 ± 0.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gils et al. [93]</td>
<td>elastic $\alpha$ scattering</td>
<td>0.175 ± 0.050</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ray [94]</td>
<td>elastic $\bar{p}$ scattering</td>
<td>0.229 ± 0.050</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Clark et al. [95]</td>
<td>elastic $p$ scattering</td>
<td>0.103 ± 0.040</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shlomo et al. [96]</td>
<td>elastic $p$ scattering</td>
<td>0.10 ± 0.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gibbs et al. [97]</td>
<td>elastic $\pi$ scattering</td>
<td>0.11 ± 0.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>combined results</td>
<td></td>
<td>0.129 ± 0.053</td>
<td>0.215 ± 0.012</td>
</tr>
</tbody>
</table>

| $^{207}$Pb | Starodubsky et al. [99]    | elastic $p$ scattering | 0.186 ± 0.041       | 0.175 ± 0.023             |
| $^{208}$Pb | Starodubsky et al. [99]    | elastic $p$ scattering | 0.197 ± 0.042       |                           |
|           | Ray [94]                   | elastic $\bar{p}$ scattering | 0.16 ± 0.05         |                           |
|           | Clark et al. [95]          | elastic $p$ scattering | 0.119 ± 0.045       |                           |
|           | Zenihiro et al. [98]       | elastic $p$ scattering | 0.211 ± 0.063       |                           |
|           | Friedman [92]              | elastic $\pi^+$ scattering | 0.11 ± 0.06         |                           |
|           | Friedman [92]              | elastic $\pi$ scattering | 0.15 ± 0.08         |                           |
|           | combined results           |                 | 0.159 ± 0.041       | 0.179 ± 0.023             |

Symmetry Energy

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$S(\rho)$ from Combined Constraints

- IAS + $\Delta r_{np}$ constraints
- Extrapolation
- ska25s20

Symmetry Energy

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Comparison to Microscopic Calculations

- APR: V18 + UVIX
- BHF: V18 + UVIX
- DBHF: Bonn A
- BHF: Bonn B + micro 3N
- BHF: V18 + micro 3N
- Extrapolation
- IAS + $\Delta r_{np}$ constraints
- Extrapolation
Symmetry Energy at $\rho_0$?

\[ S(\rho) = a_a^V + \frac{L}{3} \frac{\rho - \rho_0}{\rho + 0} + \ldots \]
Conclusions

- Symmetry-energy term weakens as nuclear mass number decreases: from $a_a \sim 23$ Mev to $a_a \sim 9$ MeV for $A \lesssim 8$.

- For $A \gtrsim 25$, $a_a(A)$ may be fitted with $a_a^{-1} = (a_a^V)^{-1} + (a_a^S)^{-1} A^{-1/3}$, where $a_a^V \approx 35$ MeV and $a_a^S \approx 10$ MeV.

- Weakening of the symmetry term can be tied to the weakening of $S(\rho)$ in uniform matter, with the fall of $\rho$.

- Including skin sizes, significant, $\lesssim \pm 1.0$ MeV, constraints on $S(\rho)$ at densities $\rho = (0.04-0.13)$ fm$^{-3}$.

- Around $\rho_0$: strongly correlated $a_a^V = (30.2-33.7)$ MeV and $L = (35-70)$ MeV.

To do: Dedicated Skyrme interactions.

PD&Lee arXiv:1307.4130  Thanks: Joe & NSF PHY-1068571
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$a_a(A) = \frac{A}{4} \frac{E^*_{IAS}}{\Delta T^2}$

IAS data: Antony et al. ADNDT66(97)1

Lines: fits to $a_a(A)$ assuming *volume-surface competition* analogous to that for $E_1$. ??Fundamental knowledge??

$A_a^{-1} = (a_a^V)^{-1} + (a_a^S)^{-1}A^{-1/3}$:
- $a_a^V = 34.8\text{MeV}$
- $a_a^S = 9.7\text{MeV}$
- $a_a^V = 35.0\text{MeV}$
- $a_a^S = 10.0\text{MeV}$
Stiffness of the Symmetry Energy

\[ S \propto \rho^\gamma \]

\( \rho \) (fm\(^{-3}\))

\( \gamma \)

IAS + \( \Delta r_{np} \) constraints

extrapolation
Robustness of Macroscopic Description?

SHF: IAS + $\Delta r_{np}$

$\alpha_a$ (MeV)

$\alpha_a^S$ (MeV)

fit

$\sim/2$ matter

Symmetry Energy

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Constraints at $\rho_0$

- $1.3 < |\Delta a_a|/\delta a_a < 1.5$
- $1 < |\Delta a_a|/\delta a_a < 1.3$
- $|\Delta a_a| < \delta a_a$
- $\Delta \chi^2_{\Delta r}/(\chi^2_{\Delta r})_{\text{min}} < 1/N_s$

SHF

SHF w/interpolation

IAS

IAS + $\Delta r_{np}$